Estimating Automotive Elasticities from Segment Elasticities and First Choice/Second Choice Data

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June 15, 2006

Abstract

Of the share lost to one product because of a price change, diversion fractions are the fractions of that lost share going to each of the other products. This paper expresses product cross-elasticities in terms of diversion fractions and a scaling factor.

Since the automotive market includes more than 200 products, time-series data is insufficient for estimating all elasticities. Instead this paper estimates automotive elasticities by specifying the diversion fractions using cross-sectional first choice/second choice data and estimating the remaining scaling factor and own-elasticities using more aggregate elasticities estimated from time-series.

1 INTRODUCTION

1.1 Cross-sectional Data Constraining Time-Series Models

Knowing product demand-price elasticities(Hagerty, Carman and Russell,1988; Hauser,1988; Tellis,1988) is central to many corporate product planning and pricing decisions. The standard time-series approach for estimating elasticities specifies product demand as a parameterized function of price and estimates these parameters using time-series data. Because of the complicated patterns of cross-price interactions among different products, a realistic demand model often involves many more parameters than can be estimated with existing time series data. This is especially true when non-stationarity and multicollinearity are present.

This problem of insufficient data can be solved by placing restrictions on the cross-price interactions in a demand model[1].

[1] Another approach, the hedonic, transforms the problem of estimating many product demand functions to the problem of estimating a smaller number of attribute demand function.

Thus products might be sorted into segments using cross-sectional data with the product demand models constrained to satisfy logit assumptions within and across each segment(Kamakura and Russell,1989; Russell and Bolton,1988). But defining segments for which the logit assumptions are realistic is nearly impossible for some markets(e.g., the automotive.)

To develop an alternative approach to imposing restrictions on demand models, note that all product elasticities are deducible from:

1. how much share a product loses when its own price increases (the own-elasticity),

2. the fraction of that lost share diverted to various other products (the diversion fraction.)

This diversion fraction is frequently independent of the amount of the price change and only a function of the cross-sectional similarity between different products(Bordley,1985). Furthermore, as this paper's first section shows, standard economic conditions imply that all income-compensated demand-price cross elasticities within a given market are deducible from diversion fractions and a single scaling factor.

Hence a demand model could also be restricted by specifying its diversion fractions from cross-sectional data prior to estimating the model from time-series. For example, consider the Rotterdam demand model(Deaton and Muellbauer,1986) on an economy with n products. Specifying the diversion fractions prior to estimating the model on time-series would reduce the number of parameters requiring estimation from $n(n+2)$ (with $n(n+1)$ parameters corresponding to demand-price elasticities) to $2n+1$, a substantial reduction in the time-series task.

1.2 An Automotive Application

This paper will focus on pricing in the automotive industry. Since vehicles vary on many dimensions: size, paint, engine, interior upholstery etc., there are many ways of defining what constitutes a distinct automotive 'product.' For the purposes of competitive pricing, the definition of what constitutes a distinct product must distinguish between

- vehicles made by different vehicle manufacturers,
- vehicles with significantly different marginal costs,
- vehicles targetted at different kinds of customers.

We therefore define a distinct product in terms of two factors:

For discussion of the hedonic approach and some criticisms, see Berndt(1990), Rosen(1974), Mendelsohn(1984).

- the vehicle's platform (which specifies certain essential manufacturing attributes of the vehicle)
- the marketing division responsible for selling the vehicle (which specifies the vehicle's target buyer group and its manufacturer.)

Each specific combination of platform and division will constitute what we call a distinct 'product' (or nameplate.) (Some examples include the Chevrolet Corsica, the Pontiac LeMans, the Honda Accord etc.) Since many manufacturers own several divisions (e.g., GM's North American operations include the Chevrolet, Pontiac, Oldsmobile, Buick, Cadillac, GMC Truck and Saturn divisions) and produce several platforms, there are more than 200 distinct products.

Typically these products are grouped into seven segments(e.g., Economy, Small, Compact, Midsize, Large, Luxury and Sporty) not satisfying logit-like assumptions. Thus the large car segment consists of the Chevrolet Caprice, the Pontiac Parisienne, the Olds Delta 88, the Buick LeSabre, the Ford Crown Victoria, the Ford Grand Marquis and the Chrysler New Yorker. Automotive economists typically estimate a time-series model over the segments (which specifies seven segment own elasticities and one market own elasticity.) Marketing analysts then 'explode' these aggregate forecasts into specific forecasts about how the more than 200 products within each segment will fare, generally using cross-sectional data on product diversions.

This division of responsibility motivates the following variation on the methodology previously discussed:

1. Estimate diversion fractions from cross-sectional data, specifically data on customer first and second choice product preferences,
2. Estimate the scaling factor and product own-elasticities using the segment elasticity and market own-elasticity estimates.

This paper uses this technique to estimate more than 40,000 product elasticities.

2 ELASTICITIES FROM DIVERSION FRACTIONS & A SCALING FACTOR

2.1 The Critical Theorem

Let q_i, p_j and Y denote product i's demand, product j's price and income respectively. Let $w_i = p_i q_i$. Define s_{ij} and the income-compensated elasticity, e_{ij}, by

$$s_{ij} = \frac{\partial q_i}{\partial p_j} + q_j \frac{\partial q_i}{\partial Y}, \qquad e_{ij} = \frac{p_j}{q_i} s_{ij} \tag{1}$$

Of the sales revenue switching out of product j given a marginal increase in j's price, define the **diversion fraction**, i.e., the income-compensated fraction switching to product i versus any other product in the market $M = \{1...n\}$, by:

$$f_{ij} = \frac{w_i e_{ij}}{\sum_{k \neq j, k \in M} w_k e_{kj}} \tag{2}$$

Define $w_M = \sum_{j \in M} w_j$ and define the intramarket cross-elasticity, E_{MM}, by

DEFINITION 1: $E_{MM} = \frac{\sum_{j,i \in M, i \neq j} w_i e_{ij}}{w_M}$

Slutsky Symmetry, (i.e., $s_{ij} = s_{ji}$, for all i, j), need not apply for aggregate demand models. This paper therefore assumes that a limited version of Slutsky Symmetry holds between products in different markets, i.e.,

ASSUMPTION 1 : For all $i \in M$,

$$\sum_{k \notin M} w_i e_{ik} = \sum_{k \notin M} p_i p_k s_{ik} = \sum_{k \notin M} p_i p_k s_{ki} = \sum_{k \notin M} w_k e_{ki}$$

Given Assumption 1, the critical Theorem is:

THEOREM 1: Let $(\pi_1 ... \pi_n)$ solve:

$$\begin{aligned} \sum_{j \in M, j \neq i} f_{ij} \pi_j &= \pi_i \\ \sum_{j \in M} \pi_j &= 1 \end{aligned}$$

Then

$$\begin{aligned} w_j e_{jj} + \sum_{k \notin M} w_k e_{kj} &= -\pi_j w_m E_{MM} \\ w_i e_{ij} &= f_{ij} \pi_j w_M E_{MM} \end{aligned}$$

PROOF: The budget constraint(see Deaton and Muellbauer,1986,pg.46) implies

$$\sum_{k \neq j, k \in M} w_k e_{kj} + w_j e_{jj} + \sum_{k \notin M} w_k e_{kj} = \sum_k p_k p_j s_{kj} = 0 \tag{3}$$

and the economic homogeneity condition implies

$$w_i e_{ii} + \sum_{j \notin M} w_i e_{ij} + \sum_{j \neq i, j \in M} w_i e_{ij} = \sum_j p_i p_j s_{ij} = 0 \tag{4}$$

Substituting (3) in (2) gives

$$w_i e_{ij} = -f_{ij}(w_j e_{jj} + \sum_{k \notin M} w_k e_{kj}), \quad i \neq j \tag{5}$$

Substituting (5) in (4) gives

$$w_i e_{ii} + \sum_{j \notin M} w_i e_{ij} - \sum_{j \neq i, j \in M} f_{ij}(w_j e_{jj} + \sum_{k \notin M} w_k e_{kj}) = 0 \tag{6}$$

Using Assumption 1 in (6) gives

$$w_i e_{ii} + \sum_{k \notin M} w_k e_{ki} - \sum_{j \neq i, j \in M} f_{ij}(w_j e_{jj} + \sum_{k \notin M} w_k e_{kj}) = 0 \tag{7}$$

If all automotive products sustained the same marginal percentage price increase, then the total sales revenue switching from one vehicle to another would be proportional to $\sum_j w_j e_{jj} + \sum_{k \notin M} w_k e_{kj}$. To focus on the fraction of these diversions associated with sales revenue switching out of product j, define

DEFINITION 2 : $\pi_j = \frac{w_j e_{jj} + \sum_{k \notin M} w_k e_{kj}}{\sum_{j \in M} [w_j e_{jj} + \sum_{k \notin M} w_k e_{kj}]}$

Thus π_j measures how elastic product j's share demand is relative to that of other automotive products. Definition 2 implies:

$$\sum_{j \in M} \pi_j = 1 \tag{8}$$

Substituting Definition 2 into (7) gives

$$\pi_i = \sum_{j \neq i, j \in M} f_{ij} \pi_j \qquad i \in M \tag{9}$$

Thus the homogeneity and aggregation conditions, when written in terms of diversion fractions, are equivalent to a Markov Process with the diversion fractions as transition probabilities and π_j as the Markov stationary probability.

Using (4) in Definition 1 gives

$$w_M E_{MM} = - \sum_{j \in M} (w_j e_{jj} + \sum_{k \notin M} w_k e_{kj})$$

Substituting into Definition 2 then gives

$$w_j e_{jj} + \sum_{k \notin M} w_k e_{kj} = -\pi_j w_M E_{MM} \tag{10}$$

Equations (5) and (10) then imply

$$w_i e_{ij} = -f_{ij}(w_j e_{jj} + \sum_{k \notin M} w_k e_{kj}) = f_{ij} \pi_j w_M E_{MM} \tag{11}$$

which proves the Theorem.

2.2 Own-Elasticities from Diversion Fractions and a Scaling Factor

If diversion fractions for all products were known, then the market would include all products and equations (10) and (11) become:

$$\begin{aligned} w_j e_{jj} &= -\pi_j w_M E_{MM} \\ w_i e_{ij} &= f_{ij} \pi_j w_M E_{MM} \end{aligned}$$

so that product own-elasticities are also deducible from diversion fractions and a single scaling factor.

If all products were strongly separable and if e_{iY} was product i's income-elasticity, then

$$\begin{aligned} f_{ij} &= \frac{w_i e_{ij}}{\sum_{k \neq j} w_k e_{kj}} = \frac{w_i e_{iY} w_j e_{jY}}{\sum_{k \neq j} w_k e_{kY} w_j e_{jY}} = \frac{w_i e_{iY}}{1 - w_j e_{jY}} \\ \pi_i &= \frac{w_i e_{iY}(1 - w_i e_{iY})}{\sum_k w_k e_{kY}(1 - w_k e_{kY})} \end{aligned}$$

Hence the elasticities become

$$\begin{aligned} w_j e_{jj} &= -w_j e_{jY}(1 - w_j e_{jY})K \\ w_i e_{ij} &= w_i e_{iY} w_j e_{jY} K \end{aligned}$$

with K a constant. This reproduces Frisch's results(1959).

3 ESTIMATING DIVERSION FRACTIONS

The author had access to a quarterly sample of 40,000 car buyers[2] who were asked to list the car they bought (their first choice) and the car they would have bought if their first choice car was made unavailable(their second choice.) This data can be used to estimate f_{ij} by making the assumption:

ASSUMPTION 3: The fraction of j's potential switchers who would divert to i, $\frac{q_i e_{ij}}{\sum_{m,j \in M, m \neq j} q_m e_{mj}}$, equals the fraction of j buyers with i as a second choice. As Appendix III shows, this assumption holds if the income-compensated demand model describing the economy is the ratio of multilinear functions of price, i.e., if

$$q_i = \frac{a_i + \sum_{k=1}^{n} a_{ik} g_k(p_k) + \sum_{j,k=1}^{n} a_{ijk} g_j(p_j) g_k(p_k)}{b_0 + \sum_{k=1}^{n} b_k g_k(p_k) + \sum_{j,k=1}^{n} b_{jk} g_j(p_j) g_k(p_k)} \tag{12}$$

where g_j is an arbitrary monotonically increasing function of product j's price, $a_i, a_{ik}, a_{ijk}, b_0, b_k, b_{jk}$ are constants and $a_{ijk} = b_{jk} = 0$ for $j = k$.

Some special cases of this multilinear demand model are

(I) **Logit** $q_i = a_{ii} \frac{e^{u_i}}{\sum_{k=1}^{n} b_k e^{u_k}}$ where u_i decreases with p_i, satisfies (12) with $g_i = e^{u_i}$, $a_{ii} = a_{ii}$, $b_k = \sum_i a_{ik}$ and all other parameters equal to zero.

[2]In a four page survey sent out to 40,000 buy car buyers a few months after they purchased their new car, buyers are asked to list the car they bought. They are also asked:'Suppose the car you purchased was not offered for sale. What other vehicle (car or truck) make and model would you have purchased instead?' The sample was stratified to ensure that a statistically significant number of buyers of each of more than 200 car and truck nameplates were sampled. Both domestic, Asian and European makes were sampled. This survey has been collected for more than ten years with no alteration in the first choice/second choice preference question.

(II) **Dogit** $q_i = \alpha_0 \frac{e^{u_i} + \theta_i(\sum_{k=1}^n e^{u_k})}{\sum_{k=1}^n e^{u_k}(1+\sum_{j=1}^n \theta_j)}$ with $\theta_i > 0$ satisfies (12) with $g_j = e^{u_j}$, $a_{ii} = \alpha_0(1+\theta_i)$, $a_{ij} = \alpha_0\theta_i$ for $i \neq j$, $b_j = \sum_{i=1}^n a_{ij}$ and zero for all other parameters.
(III) **Additively Separable Sales Models:**
$q_i = \sum_j a_{ij} g_j(p_j)$ satisfies (12) with $b_k = b_{jk} = 0$.
(IV) **The Quadratic Cost Function:** If p_0 is a numeraire, then the cost function, $c(p_0, p_1...p_n) = p_0 \sum_{ij} a_{ij}[\frac{p_i}{p_0} - a_i][\frac{p_j}{p_0} - a_j] + \alpha u$, implies a Hicksian demand model linear in price for all products other than product 0(which forms a special case of III).
(V) **A Multilinear Generalized Extreme Value Model:**
$q_i = \frac{e^{u_i} G_i(e^{u_1}, e^{u_2}, ...e^{u_n})}{G(e^{u_1}, ...e^{u_n})}$ with $G(e^{u_1}, ...e^{u_n}) = \sum_{k=1}^n b_k e^{u_k} + \sum_{j,k=1}^n b_{jk} e^{u_j} e^{u_k}$ with $b_{jj} = 0$
satisfies (12) with $a_{ii} = b_i, a_{iik} = b_{ij}$ and all other parameters equal to zero.

For those problems in which first choice/second choice data is lacking, a more general way of estimating diversion fractions involves constructing multiattribute product positioning maps(Hotelling,1929;Economides,1986; Hauser and Shugan,1983) from cross-sectional information on product attributes[3]. In this case, Assumption 3 is unnecessary.

Given f_{ij}, $(\pi_1...\pi_n)$ can be deduced from solving (8) and (9) by standard techniques[4] in Markov Chains(Ross,1972). Then (11) specifies all cross elasticities within the market up to a scaling factor, $w_M E_{MM}$.

[3]To construct multiattribute product positioning maps, a space of vectors, with each vector having r arguments representing utility weights for each of r product attributes, is defined. The sales of a product are distributed among all the vectors whose utility weights would assign that product higher utility than any other product. (A variation of this technique distributes first choice/second choice frequencies.) Once the number of people with various utility weights is specified, diversion fractions are easily deduced. (Such maps could, in principle, be used to infer all elasticities but the author prefers to use them only to infer diversion fractions.)

[4]Since the $n+1$ equations are linearly dependent, drop any single equation other than equation (8), and solve the remaining n equations for the π_j's by matrix inversion. Since the deleted equation is just a linear combination of the remaining n equations, this solution to the n remaining equations necessarily satisfies the one deleted equation.

4 ESTIMATING SCALING FACTORS & OWN ELASTICITIES

4.1 Estimating the Scaling Factor

The automotive market is divided into segments for which segment own elasticities, E_I, and an overall market own elasticity, E_M, were estimated[5]. The segments were constructed so that market and segment own-elasticities are weighted averages of product elasticities(Wold,1953), i.e.,

$$E_M = \frac{\sum_{i,j\in M} p_i q_i e_{ij}}{w_M}, \quad w_M = \sum_{i\in M} p_i q_i \tag{13}$$

$$E_I = \frac{\sum_{i,j\in I} p_i q_i e_{ij}}{\sum_{i,j\in I} w_i}, \quad w_I = \sum_{i\in I} p_i q_i \tag{14}$$

This condition is plausible since competitors in each segment have historically tended to match each other's price changes. Drastic violations of this condition(e.g., E_I being a simple arithmetic average rule of the elasticities) can cause errors of as much as 25%. But empirical results (reported in section 5) seemed robust to small violations of this condition.

These definitions of segment and market own-elasticity imply

$$\sum_I w_I E_I - w_M E_M = -\sum_I \sum_{j\in I, i\notin I} w_i e_{ij} = -w_M E_{MM} \sum_{j,i\notin I(j)} f_{ij}\pi_j$$

where $I(j)$ denotes the segment containing product j. Then the intramarket cross-elasticity, E_{MM}, is computable from

$$E_{MM} = -\frac{\sum_I w_I E_I - w_M E_M}{w_M(\sum_{j,i\notin I(j)} f_{ij}\pi_j)} \tag{15}$$

[5]For the automotive example, the best estimates of E_I and E_M came from the estimation of the segment-level demand model:

$$Q_I = \sum_{IJ} a_{IJ} p_J + b_I Y$$

where a_{IJ} and b_I and parameters and Y is income. All products were aggregated into seven segments: economy, small, compact, mid-sized, large, luxury and sporty. The price index for each segment was the average retail price of each car in that segment, corrected for rebates in a given year. The sales index was the total reported sales of cars in that segment in a given year. The regression was run over ten years. To make this model estimable, it was noted that $\frac{a_{IJ}}{a_{JJ}}$ equalled the fraction of buyers in segment J who would switch to products in segment I if all products in segment J were made unavailable. This quantity can be estimated from the first choice/second choice database. Hence the actual regression only involved the parameters a_{JJ} and b_I. Income-compensated segment elasticities can be deduced from this regression. The estimated regression satisfied the standard diagnositicity checks.

Thus the total revenue switching between segments ($-(\sum_I w_I E_I - w_M E_M)$) equals the fraction of sales that would switch between segments given a price change ($\sum_{j,i \notin I(j)} f_{ij}\pi_j$) multiplied by the number of sales that would switch from one product to another given a price change ($w_M E_{MM}$).

Given (15), equation (11) specifies all cross-elasticities.

4.2 Estimating Own-Elasticities

THEOREM 2:

$$w_j e_{jj} = -\pi_j w_M E_{MM} + \frac{\sum_{k \notin M} w_k e_{kj}}{\sum_{k \notin M, j \in I} w_k e_{kj}} (w_M E_{MM} \sum_{i \notin I, j \in I} f_{ij}\pi_j + w_I E_I)$$

PROOF: See Appendix I.

Define the fraction of segment I switchers with second choices inside segment I, i.e., segment I's exclusivity, by

$$F_I = \frac{\sum_{i,j \in I, i \neq j} f_{ij}\pi_j}{\sum_{j \in I} \pi_j}$$

Define segment I's share of all sales revenue divided by its share of potential switcher sales revenue, i.e., its loyalty, by

$$L_I = \frac{\sum_{j \in I} \frac{w_j}{w}}{\sum_{j \in I} \pi_j}$$

Then the average product own-elasticity in a segment is just

COROLLARY 1: $e_I = \sum_{j \in I} \frac{w_j}{w_I} e_{jj} = E_I - \frac{w_M E_{MM} \sum_{ij \in I} f_{ij}\pi_j}{w_I} = E_I - \frac{F_I}{L_I} E_{MM}$

Thus the difference between the product's own-elasticity in a segment and the segment own-elasticity increases with the segment's exclusivity and decreases with its loyalty.

To estimate product own-elasticities, an additional assumption is needed. This paper considers two alternative assumptions:

ASSUMPTION 2(a): For $j \in M$, $\sum_{k \notin M} w_k e_{kj} = w_j e_{jY} \mu_I$

where e_{jY} is product j's income-elasticity and μ_I is a segment-specific constant. Thus each segment I is weakly separable(Deaton and Muellbauer,1986) with respect to products outside the market. Substituting in Theorem 2 gives

$$w_j e_{jj} = -\pi_j w_M E_{MM} + \frac{w_j e_{jY}}{\sum_{k \in I} w_k e_{kY}} (w_M E_{MM} \sum_{i \notin I, j \in I} f_{ij}\pi_j + w_I E_I)$$

An alternative assumption is

Table 1: ELASTICITIES BY SEGMENT

Assumption	Car Segment	Segment Elasticity(E_I)	Car Line Elasticities(e_{jj}) MIN	AVE	MAX	L_I	F_I
2(a)	Economy	-1.9	-3.3	-4.7	-8.2	0.6	.63
2(b)			-3.4		-8.1		
2(a)	Small	-1.7	-1.9	-2.4	-3.1	0.8	.20
2(b)			-1.7		-3.4		
2(a)	Compact	-2.0	-2.1	-3.0	-4.9	0.9	.37
2(b)			-2.2		-4.7		
2(a)	Midsize	-2.3	-2.3	-3.3	-4.6	1.1	.48
2(b)			-2.6		-4.2		
2(a)	Large	-3.0	-3.1	-3.8	-4.3	1.3	.42
2(b)			-3.5		-4.0		
2(a)	Luxury	-2.4	-3.2	-3.7	-5.3	1.4	.70
2(b)			-3.4		-4.5		
2(a)	Sporty	-3.4	-2.6	-4.2	-6.5	1.3	.39
2(b)			-3.4		-5.3		

Market Own(E_M): -1.0   Market Cross(E_{MM}): 2.6   Average Car Own: -3.6

ASSUMPTION 2(b): For $j \in M$, $\frac{\sum_{k \notin M} w_k e_{kj}}{w_j e_{jj}} = \frac{1-\mu_I}{\mu_I}$ within each segment I

In other words, the fraction of buyers who switch out of the market is the same for all products within each segment. As Appendix II notes, this implies

$$w_j e_{jj} = \frac{\pi_j}{\sum_{j \in I} \pi_j} w_I e_I \quad j \in I$$

This formula indicates that if product j has twice the number of switchers as product k for $j, k \in I$, then the number of sales product j loses given a percentage price increase is twice the number of sales product k loses given a percentage price increase.

5 APPLICATION TO THE AUTOMOBILE MARKET

The segment own-elasticities varied by about 30% over the range of their 95% confidence interval. If the first choice/second choice data describe the population of car buyers exactly, then these confidence intervals imply confidence intervals over the product elasticity estimates. Table 1 lists the estimates under both assumptions 2(a) and under 2(b). The loyalty index is higher for higher priced products (as would be expected since higher income buyers are less price-sensitive). The economy and luxury car segment are characterized by great exclusivity, i.e., economy car buyers do not view non-economy cars as close substitutes, luxury car buyers do not view non-luxury cars as close substitutes. This, also, is what would be expected. The combined effect of low loyalty and high exclusivity causes the demand for cars in the economy segment to be much more elastic than the economy segment own-elasticity. But note that luxury car buyers are not, contrary to popular belief, substantially less price-sensitive than other buyers. This reflects how competitive the luxury car market has become.

Income-uncompensated elasticities can be estimated from these elasticities if product income elasticities are known. Bordley and McDonald(1993) estimate income-elasticities from the marketshare of each vehicle within various income groups and the population income distribution by:

1. computing how much the number of people in each income bracket changes when all incomes increase by the same marginal fraction,
2. multiplying a product's sales per customer in each income bracket by this change[6] to get a product's sales change in each income bracket,
3. summing over all income brackets and dividing by the product's original sales to get the income-elasticity.

Bordley and McDonald(1993) showed that this technique leads to very commonsensical automotive income-elasticity estimates.

6 CONCLUSIONS

This paper developed and illustrated a method for estimating income-compensated demand-price elasticities by:

- decomposing elasticities into diversion fractions, a scaling factor and own-elasticities,
- estimating the diversion fractions from cross-sectional data (specifically first choice/second choice data),
- estimating the scaling factor and own-elasticities from time-series data (specifically segment and market own-elasticities.).

This method led to very intuitive estimates of more than 40,000 demand-price elasticities for the automotive industry.

[6]Thus a product's sales per customer in each income bracket is assumed not to change.

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APPENDIX I:PROOF OF THEOREM 2

Using E_{MM} in the results of Theorem 1 gives

$$w_j e_{jj} = -\pi_j w_M E_{MM} - \sum_{k \notin M} w_k e_{kj} \tag{16}$$

Given the definition of E_I,

$$\sum_{j \in I} w_j e_{jj} = w_I E_I - \sum_{ij \in I, i \neq j} w_i e_{ij} = w_I E_I - \sum_{ij \in I, i \neq j} f_{ij} w_M E_{MM} \tag{17}$$

Using (16) and (17) gives

$$\sum_{j \in I, k \notin M} w_k e_{kj} = -w_M E_{MM} \sum_{i \notin I, j \in I} f_{ij} \pi_j - w_I E_I \tag{18}$$

so that

$$w_j e_{jj} = -\pi_j w_M E_{MM} + \frac{\sum_{k \notin M} w_k e_{kj}}{\sum_{k \notin M, j \in I} w_k e_{kj}} (w_M E_{MM} \sum_{i \notin I, j \in I} f_{ij} \pi_j + w_I E_I) \tag{19}$$

APPENDIX II

Theorem 1 shows that

$$w_j e_{jj} = -\pi_j w_M E_{MM} - \sum_{k \notin M} w_k e_{kj} \tag{20}$$

Given Assumption 2(b),

$$\begin{aligned} w_j e_{jj} &= -\mu_I \pi_j w_M E_{MM} \Rightarrow \\ w_j e_{jj} &= -\frac{\pi_j}{\sum_{j\in I} \pi_j} \sum_{j \in I} w_j e_{jj} \Rightarrow \end{aligned}$$

$$w_j e_{jj} = \frac{\pi_j}{\sum_{j\in I} \pi_j}(w_I E_I \quad - \quad w_M E_{MM} \sum_{i\in I, j \in I} f_{ij}\pi_j) = \frac{\pi_j}{\sum_{j \in I}\pi_j} w_I e_I$$

APPENDIX III:JUSTIFYING ASSUMPTION 3

Consider the class of models:

$$q_i = \frac{a_i + \sum_{k=1}^n a_{ik} g_k(p_k) + \sum_{j,k=1}^n a_{ijk} g_j(p_j) g_k(p_k)}{b_0 + \sum_{k=1}^n b_k g_k(p_k) + \sum_{j,k=1}^n b_{jk} g_j(p_j) g_k(p_k)} \tag{21}$$

where g_j is an arbitrary monotonically increasing function of product j's price, $a_i, a_{ik}, a_{ijk}, b_0, b_k, b_{jk}$ are constants and $a_{ijk} = b_{jk} = 0$ for $j = k$.

First note that

$$\frac{\partial q_i}{\partial p_j} = g'(p_j) \frac{a_{ij} - q_i b_j + \sum_k g_k(a_{ijk} - q_i b_{jk})}{b_0 + \sum_k b_k g_k + \sum_{jk} b_{jk} g_j g_k}$$

Thus

$$\frac{\partial q_i/\partial p_j}{\partial q_j/\partial p_j} = \frac{a_{ij} - q_i b_j + \sum_k g_k(a_{ijk} - q_i b_{jk})}{a_{jj} - q_j b_j + \sum_k g_k(a_{jjk} - q_i b_{jk})} \tag{22}$$

Now let $N_i = a_i + \sum_k a_{ik} g_k + \sum_{jk} a_{ijk} g_j g_k$ and $D = b_0 + \sum_k b_k g_k + \sum_{jk} b_{jk} g_j g_k$ so that $q_i = \frac{N_i}{D}$. Then the demand for product i if j became unavailable is

$$\frac{N_i - a_{ij} g_j - \sum_k a_{ijk} g_k g_j}{D - b_j g_j - \sum_k b_{jk} g_k g_j}$$

Subtracting off $q_i = \frac{N_i}{D}$ gives the change in demand for i when j becomes unavailable. This gives

$$g_j \frac{a_{ij} - q_i b_j + \sum_k g_k(a_{ijk} - q_i b_{jk})}{D - g_j(b_j + \sum_k b_{jk} g_k)}$$

Dividing this by the change in j demand when j becomes unavailable gives

$$\frac{a_{ij} - q_i b_j + \sum_k g_k (a_{ijk} - q_i b_{jk})}{a_{jj} - q_j b_j + \sum_k g_k (a_{jjk} - q_j b_{jk})} \tag{23}$$

This is just the fraction of j buyers who would switch to i if j became unavailable. By inspection (22) and (23) are equal, proving the result.