Determining the Appropriate Depth and Breadth of a Firm's Product Portfolio

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Abstract

Some firms have broad product lines; others have lean product lines. To determine the appropriate number of entries in a specific firm's product line, this paper develops a model which balances the benefits of increased revenue from a broad product line against production and engineering costs. Two innovations were central in the development of this model:

(1) Redefining how products are scored on various product attributes so that attribute scores vary normally across the population of products. This rescaling allowed us to develop a variant of the logit model which discounts the sales of a product portfolio consisting of highly similar products. This logit model is formally equivalent to a heterogeneous logit model with normally distributed preferences and normally distributed ratings on product attributes

(2) Redefining how the number of entries in a product portfolio was calculated in order to discount the significance of entries which are highly similar to existing products

We also introduced the notion of a centroid time in order to more easily adjust sales and total development costs for product lifecycle and investment lifecycle effects.

These redefinitions allowed us to model a firm's profit as a simple function of its effective number of product entries, the effective number of competitor entries, the total sales in the segment, variable profits adjusted for capacity constraints and product development costs. This leads to a simple expression for the profit-maximizing number of effective entries, both when competitor portfolios are fixed and when competitors dynamically adjust their portfolios. We illustrate how to estimate and apply the model on a realistic example.

1.INTRODUCTION

Product proliferation is widespread in many industries. It has two main advantages:

(1) Highly diverse product lines allow firms to satisfy the needs and wants of heterogeneous consumers more precisely (Lancaster,1979; Connor,1981; Quelch & Kenny,1994).
(2) Highly diverse product lines can also deter new firms from entering the market (Schmalensee,1978; Brander & Eaton,1984; Bannanno,1987) which allows remaining firms to charge higher prices(Benson,1990; Putsis,1997).)

For these reasons, Crest and Colgate have more than *35* types of toothpaste.

But despite the benefits of a broad product line, some firms successfully pursue the opposite strategy of having fewer higher quality entries of broader appeal. This strategy of having narrower product lines likewise has advantages:

(1) A narrow product line allows the firm to have lower per unit production costs when scale economies are present (Baumol et al, 1982).
(2) A narrow product line can lead to lower design costs, lower inventory holding costs and reduced complexity in assembly (Lancaster, 1979; 1990; Moorthy,1984).

Thus some PC manufacturers have trimmed their product lines (Putsis & Bayus, 2001).

The success of these two very different strategies emphasizes how the optimal number of entries in a firm's portfolio depends not only upon the firm's market but also on firm-specific factors like the firm's cost structure. This motivates this paper's focus on developing a model of both the firm and its market in order to specify the optimal number of entries for the firm. Our model is designed to retain the simplicity of earlier models (e.g., Hauser & Gaskin, 1984; Reddy, Holak & Bhatt, 1994; Sullivan, 1992; Easterfield, 1964) while being more realistic.

Specifying the optimal number of product entries is complicated by the fact that different firms define product lines differently. One firm might view two physically distinct, but highly similar, items as variants of the same basic product entry; a second firm might view these items as distinct entries. Hence there's considerable variation in how the standard definition of a distinct product entry (Kotler, 1994) is applied

Instead of relying on firms to apply their own firm-specific criteria in defining how many product entries they have (Connor,1981), this paper constructs our own definition of the *effective* number of entries in a firm's portfolio. In constructing this definition, we will be guided by Kekre and Srinivasan (1990)'s observation that product market share typically increases with the number of product entries. Specifically we will define the effective number of entries so that a firm's market share equals its expected share of the effective number of entries in the industry.

The third section of the paper links the effective number of product entries to overall profit. Overall profit is commonly written as the difference between variable profits and investment where variable profits are the product of marginal profit per unit and unit sales. To relate overall profit to the effective number of product entries, we:

(1) Write unit sales as the product of industry sales and the firm's market share
(2) Write market share as the effective number of entries from the firm divided by the effective number of entries in the industry
(3) Write the firm's investment as proportional to its effective number of entries. (There is empirical evidence supporting this specification.)
(4) Adjust marginal profit to include the costs of reserving production capacity

This profit model has several implications: As the firm increases its effective number of entries, market share (and variable profit) increase at a diminishing rate while development costs increase at a constant rate. Hence overall profits will initially rise as the effective number of entries increases, reach some maximum level, and then decrease.

The fourth section solves for the optimal effective number of product entries both when competitor portfolios are fixed and when competitors dynamically readjust their portfolios. We find that the optimal number depends on the ratio of variable profits per unit of market share to development costs per entry. Our fifth section discusses how this model was applied to a real problem.

2.Measuring the Number of Effective Entries

(2.1) The Logit Model

This section defines the effectiveness of an entry so that the market share of an entry equals its effectiveness divided by the sum of the effectiveness of all entries in the market. Given this definition, every model of market share implies an effectiveness measure (and every effectiveness measure implies a model of market share). We use this effectiveness measure to compute the effective number of entries in a firm's portfolio.

We first derive the effectiveness measure for the multinomial logit model (Ben-Akiva, M. & S.Lerman, 1979; Court, 1939; Lancaster,1966; Griliches,1972; Quandt,1968 & Rosen,1974; Shocker & Srinivasan,1974). This model presumes

Assumption 1: *A customer chooses the product of maximum actual utility. The actual utility of the product is the sum of its observed utility, u_i, and a double exponentially distributed error term.*

We model customers as evaluating each product based on its performance on multiple attributes so that

Assumption 2: *The observed utility is a weighted average of how the product scores on each attribute and the importance the customer assigns to that attribute.*

Assumptions 1 and 2 lead to a formula for the probability of an individual buying product i. Assuming that all buyers have the same importance weights implies a logit model of market share. In this model, the effectiveness of product i is an exponential function of u_i.

(2.2) Three Families of Generalizations of the Logit Model

Because the logit market share model is too simplistic for many applications, there's been considerable work on generalizing this model. This subsection briefly reviews three classes of possible extensions (Kaul and Rao,1995).

The first class of extensions (Meyer & Johnson, 1995) supplements the observed product utility, u_i, with a term, $S(T|i)$, measuring the perceived proximity between i and all other products in the choice set. Meyer & Kahn (1990), Batsell & Polking(1985) and Cooper & Nakanishi(1983) present different ways of specifying $S(T|i)$. Given this family of `Meyer-Johnson' demand models, the effectiveness of product i is an exponential function of $u_i + S(T|i)$.

The second class of extensions, the ideal point model (Carroll,1972; MacKay, Easley & Zinnes,1995; DeSoete, Carroll & DeSarbo,1986), redefines u_i to be inversely proportional to some measure of the distance between product i's score on various attributes and the scores of an `ideal point' on those attributes. Making the ideal point infinite (Kamakura, 1986) leads to vector models where u_i is consistent with Assumption 2 (Carroll,1980). Anti-ideal point models (Carroll,1972; DeSarbo & Rao, 1986) write product i's effectiveness as proportional to the distance between product i's score on various attributes and the scores of the anti-ideal product. (In most applications, the distance measure is either the weighted squared difference or the weighted absolute difference between attribute scores.)

The third class of extensions returns to the definition of u_i given by Assumption *2* but presumes that customers differ in the importance weights they assign to different attributes. (Variations across customers in the importance weights can also include variations across customers in the standard deviation of the error term.) This class includes heterogeneous logit models (Anderson, DePalma and Thisse, 1992; Kamakura & Russell,1989) in which:

(1) The market is divided into a small number of market segments
(2) Importance weights are the same within market segments
(3) Importance weights vary between market segments

This class also includes random coefficients models (Judge et al, 1985; Longford,1995) and hierarchical Bayes choice models (Allenby, Arora and Ginter,1995; Allenby and Ross,1999; Lenk et al,1996) which typically presume that the importance weights vary continuously across the population of consumers according to a normal distribution. (In most cases, the error distribution is normally distributed in random coefficient models and double exponentially distributed in hierarchical Bayes choice models.)

These three different extensions imply different market share models and different models of product effectiveness. In order to develop a single effectiveness measure, the next section presents two assumptions that lead to a single market share model closely related to all of the models reviewed in this section.

(2.3) The Proposed Model of Market share

We follow Berry, Levinsohn & Pakes(1998) and Sudhir(2001) in assuming

Assumption 3: *The importance weights are normally distributed with mean Ew and variance-covariance matrix, V*

Thus our model is similar to the heterogeneous logit and hierarchical Bayes model in assuming that the error term is double exponentially distributed and similar to the random coefficients and hierarchical Bayes model in assuming that the importance weights are normally distributed. Assumption 3 implies that most customers attach intermediate importance to an attribute, a few customers attach very high importance to the attribute and a few customers attach very low (or even negative) importance to the attribute. (In the application of interest, we verified that this assumption was approximately true for all of our attributes.)

In regression analysis, it's common to transform input variables to make them normally distributed. (Johnson, Kotz and Balakrishnan(1997) discuss various transformations to make variables normally distributed.) In this paper, it will similarly be useful to transform product attributes so that the scores on these attributes vary normally across the population of products. (Thus on any attribute, there will be a few products with extremely low scores, a few with extremely high scores and most products with intermediate scores.) In order to make this normalizing transformation, we assume

Assumption 4:Normally Distributed Scores: *Attribute scores can be transformed so that the cumulative distribution of scores on all m attributes is described by a cumulative normal distribution with a vector of mean importances Eb and variance-covariance matrix C.*

As Appendix I shows, Assumptions 1,2,3 and 4 imply

Proposition 1: Product *i*'s market share is described by a Meyer-Johnson model where

(1) A heterogeneity discount factor is defined as the inverse of $I+CV$ with I being the identity matrix. As either product variety (represented by C) or preference heterogeneity (represented by V) increases, the discount factor shrinks.
(2) u_i is a weighted average of the scores on each attribute and that attribute's overall importance. The vector of overall importances is the vector of mean importances multiplied by the heterogeneity discount factor
(3) $S(T|i)$ is the weighted squared distance between product i's score on each attribute and the average score on each attribute. The weighting function, V^*, is V multiplied by the heterogeneity discount factor.

When some attributes are categorical (e.g., only have two possible values), Assumption 4 fails. Hence a rigorous application of our methodology requires that we eliminate categorical attributes by, for example,

(1) Replacing categorical variables with non-categorical variables
(2) Segmenting our market using these categorical variables and using our model to describe market share within each segment

In practice, it's not always necessary to eliminate categorical variables. As Appendix II shows, Proposition 1 will still be approximately true---even when Assumption 4 fails---as a second-order Taylor Series approximation of any demand model.

When V is negligible, $S(T|i)$ vanishes and this model reduces to the standard logit model. As Appendix I shows, we can define a hypothetical set of attribute scores, $b^{\#}$, such that $u_i+S(T|i)$ is the weighted squared difference (using V^* as the matrix of weights) between product i's scores on those attributes and $b^{\#}$. After performing a principal component analysis, we replace V^* by a diagonal matrix, attributes by uncorrelated factors, attribute scores by factor scores and $b^{\#}$ by $f^{\#}$. If we define the distance between any two products as the weighted squared distance between their factor scores, then $u_i+S(T|i)$ is the distance between product i and an `anti-ideal point' with a score of $f^{\#}$.

As a result, our model becomes a kind of anti-ideal point model where $f^{\#}$ is the score on the anti-ideal point. This anti-ideal point score, $f^{\#}$, is the average score of all products on that factor minus an adjustment term. The adjustment factor is the average importance of that factor divided by the variance in the importance attached to that factor across the population. Since Assumption 3 presumed that importance weights for an attribute are normally distributed, the adjustment factor is related to the proportion of customers assigning a positive importance weight to the attribute. There are three special cases:

(1) If the average importance greatly exceeds the standard deviation, then most of the population assigned a positive importance weight to the attribute. In this case, the adjustment factor is large and the anti-ideal point's score will generally be less than the score of any existing product. As a result, a product's share increases as its score on the attribute increases.

(2) If the negative of the average importance greatly exceeds the standard deviation, then most of the population assigned a negative importance weight to the attribute. In this case, the adjustment factor will be large but negative and the anti-ideal point's score will generally exceed the score of any existing product. As a result, a product's share increases as its score on the attribute decreases.

(3) If the adjustment term is small, then a significant fraction of the population assigned a positive importance weight to the attribute and a significant fraction assigned a negative importance weight. As a result, some products will score more and some will score less than the anti-ideal point. In this case, increasing the attribute score will raise market share for products above the anti-ideal point and lower market share for products below the anti-ideal point.

When all individuals assign the same importance to all products (i.e., when the variance of the importance weight is zero), the adjustment term becomes infinite and our anti-ideal point converges, using arguments from Kamakura(1986), to the conventional vector model.

It is common to treat the anti-ideal point as a variant of the ideal point model for unconventional attributes. But our assumptions lead to an anti-ideal point model which is a natural extension of the vector model in the presence of preference heterogeneity and which can handle `more is better' attributes, `less is better' attributes and bipolar attributes (Kleiss and Enke,1999).

(2.4) The Effective Number of Entries

Proposition 1 presented a market share model closely related to the heterogeneous logit, random coefficients, Meyer-Johnson and anti-ideal point models. This market share model leads to the following measure of effectiveness:

Definition (Effectiveness): *If `0' denotes the product with the highest market share across all products in the industry (e.g. the `best in class' product), then the effectiveness of product i is an exponential function of the difference between* $u_i+S(T|i)$ *and* $u_0+S(T|0)$ *where* u_i, u_0, $S(T|i)$ *and* $S(T|0)$ *were defined in Proposition 1 .*

Summing the effectiveness of all products in firm *a*'s portfolio gives the *effective number of entries,* n_a, *in firm a's portfolio.* (Intuitively, it corresponds to the number of *best in class* products that would have the same market share as firm *a*'s portfolio.) This measure reduces to the number of products in the firm's portfolio when all products are distinct and have identical utility. It will be less than this actual number when products are similar or when their quality is less than that of the best-in-class product. Summing n_a over each of the *F* firms in the industry gives the effective number of entries in the industry, which we denote by *n*. Firm *a*'s predicted market share is just n_a/n.

The next section relates n_a to the firm's overall profit.

3. Relating Profit to the Effective Number of Entries

Overall profit depends on the effective number of entries as well as on the costs of developing those entries, the time lag between when entries are developed and sold, and the marginal profit from selling each entry. This section introduces assumptions on development costs, the time lag and marginal profit. Section 4 develops and solves the resulting profit model to determine the optimal effective number of entries.

(3.1) Costs of Developing a Product

The firm incurs development costs (i.e., marketing, engineering, tooling and capital costs) for each product line it launches. These development costs increase with the number of product lines launched, with the quality of the entries being launched and with the degree to which entries differ from existing product lines (e.g., whether the entry is a minor modification of existing entries, a major modification or an entirely new product). To model these complex effects, we assume

Assumption 5: *The firm's total development spending is proportional to* n_a .

To validate the reasonableness of this assumption for the firm considered in section 5, we plotted observed development costs as a function of n_a for eight different segments in that firm's market. Applying standard statistical tests indicated that the relationship between development costs and n_a was linear.

We define K to be the incremental increase in a firm's development costs associated with an incremental increase in the effective number of entries. We obtained a crude estimate of K by dividing a firm's total development costs by the number of entries, n_a. Improving how products are positioned increases n_a and thus decreases K.

(3.2) The Speed of Product Development

Development cost spending is incurred over a long period of time beginning with the initiation of the product program and culminating with its early launch. (Indeed since the idea for a product often precedes its formal development by many years, it's often hard to determine exactly when work on a new product started.) To model how spending is distributed over time, we define

Definition (Centroid Time): *The centroid time for a cash flow is defined so that the discounted present value of the cash flow, received over some finite period of time, equals the discounted present value of receiving the entire cash flow at the centroid time.*

We define T to be the difference between the *centroid time* for the revenues from selling product and the *centroid time* for project development costs. Thus T reflects the length of time before a firm gets a return on its investment. If we treat the project as starting at the centroid time for project development costs, conventional cash discounting techniques imply that revenues must be multiplied by a discount factor, $d= 1/(1+r)^T$, where r is the corporate interest rate.

In order to more easily estimate T, we assume:

Assumption 6:

(a) The fraction of a product program's development cost spending incurred t units after the start of the program is described by a gamma distribution.

(b) The fraction of a product's total sales realized t units after the product is launched is described by a gamma distribution.

We validated Assumption 6(a) for the firm discussed in section 5 by finding that it closely described development spending patterns for five of the firm's engineering centers. Assumption 6(b) is consistent with the conventional, empirically validated, understanding of the product lifecycle (Chase et al, 2001). Appendix III estimates the time lag between revenue and investment cost from the parameters of these gamma distributions.

(3.3) Profit Margin

In many companies, it's common to assume that

Assumption 7: *Variable labor, material, shipping, warranty, depreciation and interest costs are all linear functions of expected demand.*

There are also variable costs associated with reserving capacity for building units of product. It's common practice in operations management (Chase et al, 2001) to assume:

Assumption 8: *Optimal capacity equals expected demand plus the product of a safety factor, s(F), and the standard deviation of that demand.*

This assumption is justified if demand is normally distributed. We validated this assumption for the firm considered in section 5 by reviewing the firm's historical studies of product demand. Appendix IV describes how the safety factor is computed.

Assumption 9: *The ratio of the standard deviation of demand to expected demand or s* is constant across all the products in the firm's portfolio.*

We likewise verified this assumption by reviewing internal historical studies of demand. Given Assumption 9, we can define:

Definition: Imputed Capacity cost*: The imputed capacity cost per product equals the product of cost per unit of capacity reserved and (1+s(F)s*).*

We define the variable profit associated with a product as its price less conventional marginal costs (labor, warranty, shipping, etc) and imputed capacity costs.

Our final assumption is purely for analytic convenience

***Assumption 10**: The marginal profit associated with each incremental unit of market share is unaffected by a change in the number of entries in the firm's portfolio.*

Since we did not have empirical data bearing on this assumption, a later section will discuss how the implications of our model change when this assumption is relaxed.

4. The Profit Model

The previous sections made ten assumptions about cost and sales. As we now show, these assumptions lead to a very simple formula for optimal competitive behavior.

(4.1) The `Back of the Envelope' Model of the Firm

If we treat a product program as starting at its centroid development time and if S is the total demand in the market, then our ten assumptions imply that the discounted present value of the firm's *total profit* is

$$S\, d\, \pi_a \frac{n_a}{n} - K\, n_a$$

This formula writes profit as the difference between variable profits and development costs. We define firm *a*'s R-factor by

$$R_a = \frac{d\, S\, \pi_a}{K_a}$$

The firm's *R*-factor increases as marginal profits or industry sales volume increases and decreases as development costs or the time lag, *T*, between sales and development spending increases. If $n^*_a = n - n_a$ denotes the effective number of entries competing with firm *a,* then the firm can only profitably enter the market if R_a exceeds n^*_a. As Appendix V shows, profits will initially increase as the firm adds entries until profit is maximized when the total number of effective entries in the market, n, is a geometric average of R_a and n^*_a. (By construction, the effective number of entries need not be integer.) Increasing entries beyond that point causes profits to decrease. The firm's profit becomes zero when $n = R_a$.

(4.2) Competitive Equilibrium

But as Choi, De Sarbo and Harker (1990) noted, competitors will modify their portfolios if firm *a* adds (or subtracts) entries from its portfolio. To model competitive reactions, let $R_1 \ldots R_F$ be the *R*-factors for each of the *F* firms in the industry. Define *R*, the industry *R*-factor, as a harmonic average of $R_1 \ldots R_F$. As Appendix VI shows, if all firms simultaneously specify their number of entries to maximize profit, then:

(1) The total effective number of entries in the market, *n*, will equal *R* multiplied by $(1-[1/F])$.

(2) The market share of firm *a's* competitors will equal the ratio of *R* to R_a multiplied by $(1-[1/F])$. (Firm *a's* market share can be computed as one minus the market share of its competitors.)

As a result, the key factor determining a firm's market share is the ratio of its *R*-factor to the average *R*-factor in the industry. A firm with an infinite *R*-factor will have *100%* share. When there are only two firms labeled *1* and *2*, firm *1*'s market share equals its *R*-factor divided by the sum of the *R*-factors for all firms in the industry.

No viable firm can have an *R*-factor less than the product of *R* and $(1-[1/F])$. Hence as the number of firms increases, all surviving firms will eventually have the same *R*-

value. The total number of effective entries in the market, n, will likewise converge to R. At this point, overall profit approaches zero for all firms.

(4.3) Relaxing Assumption 10

These results presume that marginal profit does not change as we change the effective number of entries. Appendix VII examines the effect of assuming that marginal profit is inversely proportional to the effective number of entries in the market. In this case, the optimal effective number of entries is smaller. (This effect is minimal when the effective number of entries in the market is large.)

5. Application to the Automotive Industry

This section presents an application of this method to the automotive industry. (Due to strategic concerns, the data has been partially disguised where indicated, while retaining realism) Our application to the automotive industry involved seven steps:

Step I: Identifying Product Attributes Satisfying Assumption 4

Standard methods were used to identify a *preliminary* list of fourteen product attributes important to customers. A conjoint study (Johnson,1987, Wittink and Cattin,1989) was conducted in which *60,000* subjects from seven major cities were shown hypothetical vehicles (e.g., a front wheel drive vehicle costing *$7500* and an all wheel drive vehicle costing *$10,000*) and were asked to specify their preference and the degree of strength of that preference for the vehicle on a ten-point scale. Conjoint analysis then estimated part-worths for each attribute level for each subject. Since one of the attributes was price, the part-worths of the other attributes can be converted into price-equivalents. The price-equivalent can be interpreted as the willingness to pay for a certain attribute level. We will refer to these willingness to pays as *preliminary.*

Assumption 4 presumes that all attributes can be transformed so that attribute scores vary normally across the population. Unfortunately four of our attributes: drive-type, transmission type, make/brand and body style could not be transformed in this way. To address this problem, we first examined the make/brand variables---which were a set of dummy variables for each distinct brand. We observed that:

(1) The empirical willingness to pay estimates for the different brand names followed a distinctive pattern: the more prestigious the brand, the higher the willingness to pay. (Thus Jaguar and Mercedes had some of the higher willingness to pays, willingness to pays for Buick and Toyota were more intermediate, Hyundai and Kia had the lowest willingness to pays.) In addition, the correlation between willingness to pay for brand-name and the average price of the brand exceeded *0.8.*

(2) We applied a principal component analysis to the *preliminary* willingness to pay data. The first and most important principal component loaded heavily on all of the brand names and not on any other variable. We interpreted this result as indicating that:

   (a) Individual buyers generally agreed on which brand names were more valuable and which were less valuable. As a result, most people agree that a Mercedes brand name is more valuable than a Toyota brand name and that a Toyota brand-name is more valuable than a Kia brand-name. (Since our data is cross-sectional, this pattern doesn't necessarily conflict with brand loyalty where past buyers of Toyota vehicles assign more value to the Toyota brand name than to the Ford brand name while past buyers of Ford vehicles assign more value to the Ford brand name than to the Toyota brand name.)

   (b) Individuals disagreed on how much they were willing to pay for more valuable brand names. In other words, individuals who buy Kia instead of Mercedes don't buy Kia because they considered the Kia brand name more valuable. Like everyone else, they recognize that the Mercedes brand name is more valuable than the Kia brand name. They buy Kia because, unlike Mercedes buyers, they are unwilling to pay the money required to get the more valuable brand name.

Given these two findings, we defined a single underlying variable called *brand reputation* and defined an entry's score on that variable to be the average willingness to pay for the brand corresponding to that entry. After introducing the *brand reputation* variable, we eliminated the brand-name dummy variables.

We then examined the variable *body style* (whose levels included small, medium, large, minivan etc.) We note that *drive-type* and *transmission-type* seemed to be strongly correlated with *body style*. Since these three variables:

(1) Did not satisfy Assumption 4
(2) Defined physical characteristics and not customer wants and needs
(3) Were generally associated with different profit margins and development costs
(4) Could not be easily replaced---as we replaced the brand-name dummy variables---by an underlying non-categorical variable like `brand reputation'

we segmented our sample by *body style* and applied our model to buyers within each *body style* segment. In this way, we eliminated these three variables.

Our remaining ten attributes were non-categorical. These were passing acceleration, repair frequency (quality), front interior width, number of features, towing capacity, workmanship, towing capacity, turning circle, fuel economy and cargo capacity.

Step II: Estimating the Effective Number of Entries in the Portfolio

The database, www.autosite.com, listed the scores of various automobiles on our ten attributes (as well as on many other attributes.) We plotted histograms of the distribution of scores on each attribute. If the distribution did not appear to be approximately a normal distribution, we used Box-Cox power transformations (specifically logarithmic and reciprocal transformations) to redefine the attribute so that its score would be approximately normally distributed across products. (For example, fuel economy---commonly measured in miles per gallon---was replaced by a measure of the number of gallons used per mile.) We then estimated the mean, *Eb*, and variance-covariance matrix, *C*, of the transformed attribute scores (within each segment). An illustrative example (for the midsized segment) is presented below

TABLE I: TRANSFORMED ATTRIBUTES

Variable	Mean	Variance Covariance Matrix	Reputation	Cargo	Quality	Work Manship	Feature	Interior Width	Fuel Economy	Horse power	Turning Circle	Towing
Reputation	2.59	**Reputation**	1.9	-.04	-15	.00	.00	.06	3.50	-17.9	442	10.2
Cargo	1.77	**Cargo**	-.04	1.4	.03	45	.90	69	-19	-148	-56	36
Quality	0.39	**Quality**	.01	.03	.18	.14	.90	.70	-.10	-.50	-0.18	-0.18
Workmanship	0.29	**Workman**	.018	.20	.30	.02	1.40	1.10	-.10	-.78	-0.24	-0.26
Feature	0.3	**Feature**	.06	.90	.09	.14	.04	.36	-.50	-2.40	-1	-.97
Interior	1.9	**Interior**	.004	.07	.07	.11	3.60	.17	-.50	-11.7	52	16.3
Fuel Economy	.22	**Fuel Economy**	-.03	-02	-.09	-.014	-.50	-.50	.22	-.14	-34	-62
Horsepower	.197	**Horse Power**	.62	-.015	-.05	.078	-2.4	-.12	-.14	.34	.06	.62
Turning Circle	.19	**Turning Cir**	.02	-.06	-.002	.024	-.10	52	-34	.06	.30	.03
Towing	.17	**Towing**	-.02	-.04	-.02	.01	-0.097	16	-62	.62	.028	.04

(Attribute scores are defined by dividing the vehicle's actual score by the score of a Subaru pickup.) Given these transformed variables, we returned to the raw database underlying our conjoint analysis and reestimated willingness to pay values for incremental changes in these transformed attributes. We assigned each customer to a segment based on their willingness to pay for each body style. For each attribute, we constructed a histogram of the importance weights across customers within a segment and verified that the histogram had the bell shape characteristic of a normal distribution. After computing the mean, *Ew*, and variance-covariance, *V*, of the importance weights within each segment, we defined V^* by multiplying V by the inverse of $I+CV$. Table II presents V^*.

TABLE II:
The Total Variance(V*) Matrix

	Reputation	Cargo	Quality	Workman ship	Fea tures	Interio r	Fuel	Acceler	Turning	Towing
Reputation	1.08	0.05	0.07	0.19	0.01	0.00	0.11	0.20	0.13	0.15
Cargo	0.02	0.81	0.67	0.81	0.74	0.99	0.16	0.19	-0.75	0.13
Quality	0.12	0.01	0.18	0.14	0.01	0.02	-0.54	0.18	0.12	1.03
Workman	0.18	0.03	0.09	0.16	0.07	0.05	0.05	1.11	0.03	0.08
Features	0.18	0.04	0.09	0.19	0.10	0.16	0.04	0.10	0.18	0.11
Interior	0.19	0.00	0.06	0.09	0.10	0.14	0.17	0.13	0.08	0.14
Fuel	0.08	0.07	0.16	0.15	0.20	0.14	0.01	0.03	0.04	0.04
Accelerat	0.17	0.11	0.20	0.07	0.01	0.02	0.15	0.01	0.07	0.19
Turning	0.03	0.07	0.05	0.16	0.05	0.03	0.15	0.16	0.07	0.14
Towing	0.00	0.06	0.20	0.06	0.07	0.16	0.02	0.10	0.04	0.11

We also defined the anti-ideal point for each segment as the difference between the mean rating on each attribute and the product of the mean willingness to pay and the inverse of V. For each product, we computed the weighted squared distance between that product's attribute scores and this anti-ideal point using V^* as the weight matrix. This allowed us to compute the product's effectiveness as proportional to an exponential function of this difference. Summing the effectiveness measure across all products in the firm's portfolio gave the effective number of entries in the firm's portfolios and in rival competitor portfolios. For the mid-sized segment, the effective number of entries in the firm's portfolio (using our disguised numbers) was *7.5* while the actual number was *15*. Hence the firm's products in the mid-sized segment were perceived as very similar.

Step III: Estimating Development Costs

To estimate development costs, we collected information on engineering, tooling and new product promotion costs and summed them to get total costs. Dividing by the effective number of entries in the firm's portfolio gave development costs per effective number of entries. Table III presents this (disguised) information.

TABLE III: DEVELOPMENT COST

	Engineering Budget (in $Billion)	Tooling Budget (in $Billion)	Ad Budget (in $Billion)	Total Development Cost (in $Billion)	Effective # of Firm's Entries	Effective # of Competitor Entries	Development Cost per effective entry (in $Billion)
LOW	1.6	1.5	0.17	3.2	5.00	16.8	0.64
MID	2.8	3.2	0.32	6.4	7.52	17.7	0.85
LARGE	0.2	1.2	0.09	1.4	3.00	1.7	0.48
LUXURY	1.1	1.4	0.42	2.9	3.94	18.3	0.75
SPORT	0.7	0.7	0.10	1.5	2.00	11.4	0.74
VANS	1.2	2.0	0.25	3.4	5.30	13.9	0.63
PICKUPS	5.4	5.6	0.39	11.4	15.50	26.1	0.73
UTILITY	2.3	3.2	0.45	5.9	8.65	12.2	0.68

Step IV: Estimating the Profit Margin for each Segment

Disguised estimates for marginal costs are listed below. Adjusting marginal profits to incorporate imputed capacity costs gives net profits. Discounting net profits to reflect the time lag between investment and revenue gives the profit margin:

TABLE IV: MARGINAL PROFIT

	Dealer Price (in $'000)	Traditional Marginal Cost(in $'000)	Capacity Cost In $'000)	Standard Deviation of Demand (in '000)	Safety Factor	Imputed Capacity Cost(in $'000)	Net Profit In $'000)	Time Delay	Interest Rate	Profit Margin In $'000)
LOW	12.6	9.3	$ 2.0	170	1.01	2.3	1.0	2.0	0.150	0.7
MID	18.5	11.9	$ 2.5	150	1.39	3.0	3.6	2.0	0.150	2.6
LARGE	20.7	12.9	$ 3.1	230	1.36	4.1	3.7	2.0	0.150	2.7
LUXURY	33.5	18.7	$ 4.6	190	1.52	6.0	8.8	2.0	0.150	6.4
SPORT	18.9	7.2	$ 2.6	350	1.73	4.2	7.4	2.0	0.150	5.4
VANS	22.8	14.0	$ 3.0	290	1.46	4.3	4.5	3.0	0.150	2.8
PICKUPS	23.8	10.0	$ 3.3	150	1.70	4.1	9.7	4.0	0.150	5.1
UTILITY	27.4	18.2	$ 4.5	190	1.19	5.6	3.7	3.0	0.150	2.3

Step V: Estimating the R Value

In Table V, we write the net present value of profit as the product of the annual export-adjusted volume, the variable profit per sale and the present value of the time period over which the product is sold (i.e, the discounted lifecycle.) Discounting for the time delay till the product is launched and dividing by development cost per entry gives the *R*-value:

TABLE V:
COMPUTING THE R VALUE

	Export-Adjusted Volume (in 'Millions)	Discounted Life Cycle	Total Sales (in Millions)	Net Profit (in $'000)	Time Delay Discount	Development Cost per effective entry (In $Million)	R-ratio
LOW	1.8	4.2	7.6	1.0	0.74	640	8.7
MID	2.7	4.3	11.5	3.6	0.74	850	36
LARGE	0.3	3.7	1.0	3.7	0.74	480	5.6
LUXURY	0.9	3.7	3.4	8.8	0.74	750	30
SPORT	0.5	4.2	2.0	7.4	0.74	740	15
VANS	1.0	4.4	4.7	4.5	0.64	630	21
PICKUPS	2.3	4.5	10.6	9.7	0.55	730	76
UTILITY	2.3	4.5	10.6	3.7	0.64	680	36

Step VI: Estimating the Required Changes in the Firm's Portfolio

Given the *R* ratio, we can compute a target number of effective entries in the industry and thus, after subtracting off the effective number of competitors, the desired number of effective entries for the firm:

TABLE VI:

OPTIMAL NUMBER OF ENTRIES

Segments	Critical Ratio	Effective Number of Competitor Entries	Optimal # of Entries in Market	Optimal Effective # of firm entries	Actual Effective # of Entries	Proposed Change	Actual # of Entries as Counted by Firm	Change in Actual # of Entries
LOW	8.7	16.8	12	0.0	5	-5.0	10	-10
MID	36	17.7	25	7.6	7.5	.07	15	0
LARGE	5.6	1.7	3	1.4	3	-1.6	2	-1
LUXURY	30	18.3	23.5	5.2	3.9	1.3	3	1
SPORT	15	11.4	12.9	1.5	2	-.5	7	-2
VANS	21	13.3	16.7	3.4	5.3	-1.9	13	-5
PICKUPS	76	26.1	44.7	18.6	15.5	3.1	6	1
UTILITY	36	12.2	21.1	8.9	8.7	.22	17	0

We used these numbers to construct the following table

TABLE VII:
CHANGES IN MARKET SHARE & PROFIT

Segments	Actual # of Effective Entries	Actual Share	Actual Profit (in $Billion)	Optimal # of Effective Entries	Optimal Share	Optimal Profit (in $Billions)
LOW	5.0	0.23	-1.9	0	0	0
MID	7.5	0.30	2.8	7.6	0.30	2.8
LARGE	3.0	0.64	0.3	1.4	0.45	0.5
LUXURY	3.9	0.18	1.0	5.2	0.22	1.1
SPORT	2.0	0.15	0.14	1.5	0.12	0.2
VANS	5.3	0.28	0.44	3.4	0.20	0.6
PICKUPS	15.5	0.37	9.5	18.6	0.42	9.7
UTILITY	8.7	0.41	4.4	8.9	0.42	4.4
			16.7			19.2

Hence the model recommended the firm not have any share in the low segment, have smaller share in the large, sports and van segment, and have higher share in the pickup segment. Hence the firm's future product programs should be adjusted so that:

(1) no new product programs are started in the low segment
(2) new product programs in the large, sports & van segments should be fewer and involve less differentiation (which decreases the effective number of entries)

(3) more new product programs with greater differentiation are introduced in the pickup segment (which increases the effective number of entries.)

Such changes will cause overall profit to increase by $*2.5* billion. (Canceling existing product lines would lead to a smaller profit increase since the costs associated with developing existing product lines cannot be recovered.)

Our model is designed to be simple while incorporating the impact of capacity costs, development cost spending, product differentiation, capital costs, competition, etc. A natural question is whether or not the model could be further simplified by eliminating some of these factors. To address this question, note that the model recommends decreases in the low car segment because of the low level of variable profits, decreases in the sports segment because of high development costs and negligible increases in the utility segment because of the large number of existing competitors in that segment. Eliminating any one of these factors from the model would have led to different predictions. Hence a more simplified model would probably have made different recommendations.

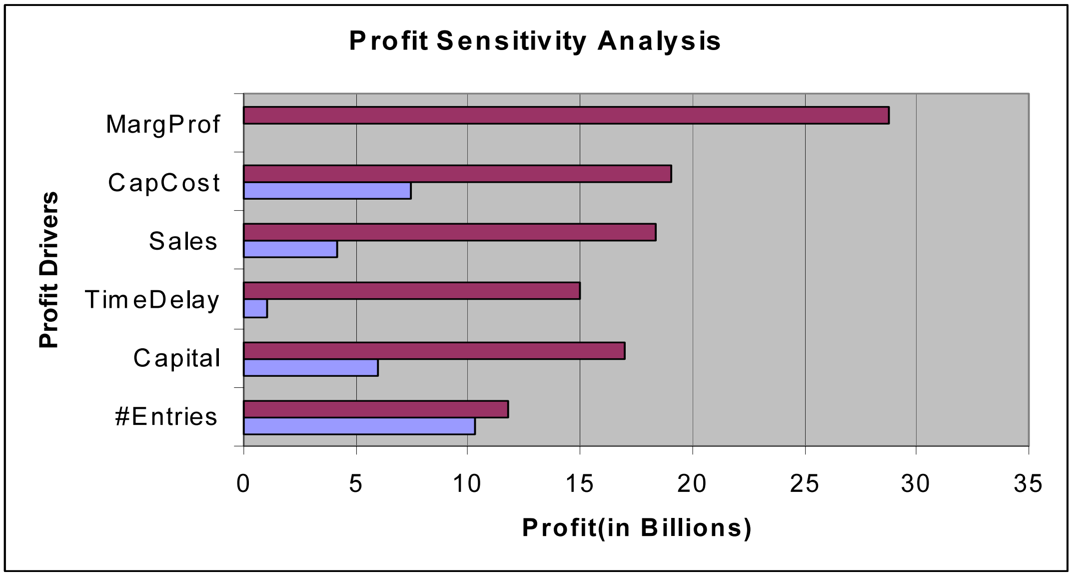
When the firm reduces its product portfolio, other firms may respond by increasing their product portfolio. Section (4.2) derives the resulting equilibrium market shares.

Step VII: Sensitivity Analysis

We conducted a sensitivity analysis examining how much profit varied as we varied the number of entries. If the number of entries are at their current level, then the following chart describes how changing different variables by *15%* impacts profit

TABLE VIII:

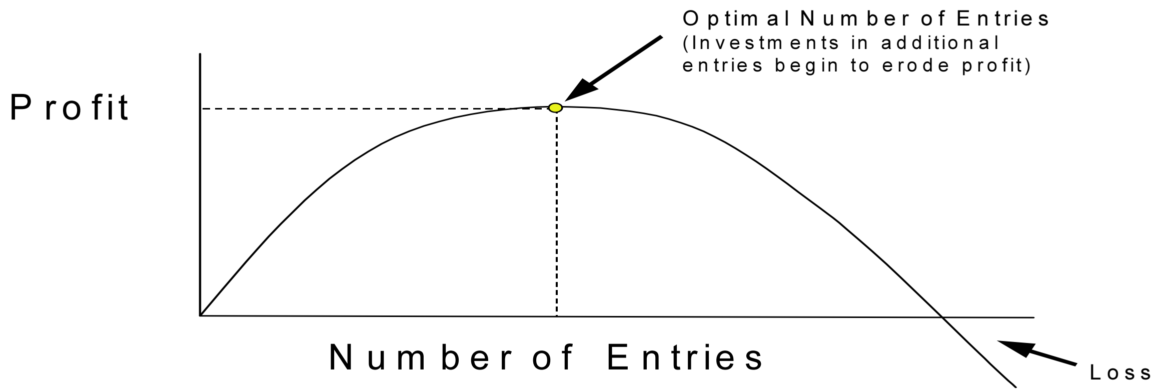
SENSITIVITY ANALYSIS



For each driver, there are two bars (where the lower bar for marginal profit is too small to be observed.). The upper bar describes the profit level when the driver is increased by *15%;* the lower bar describes the profit level when the driver is decreases by *15%.* The importance of a driver can be estimated by looking at the difference between the profit level from the upper bar and the profit level from the lower bar.

Note that marginal profit is the most important driver and causes profits to vary from *$28* billion to less than a billion. Capacity costs are next most important followed by sales, the time delay and capital costs. The number of entries is of least importance. To understand why the number of entries doesn't have that much impact, we plotted profit as a function of the number of entries and obtained the following graph:

Overall Profit as a function of the Number of Entries



The graph indicates that profitability does not vary much with the number of entries when the number of entries is close to the optimum level (although it is important when the number of entries is very far from its optimum value.) Since the actual number of entries for our firm was not very different from the optimum level, this explains why changes in the number of entries only had a small impact on profit.

6. Adjustments to the Analysis

(6.1) Corporate Average Fuel Economy Requirements (CAFÉ)

In our optimal portfolio, we stopped developing new products for the low segment. But in the automotive industry, the average fuel economy associated with all the cars sold by a firm is required to exceed a certain target level (with a similar constraint holding separately for all trucks.). Since vehicles in the low segment are very fuel-efficient and since the firm was close to the CAFÉ limit, eliminating new product development in the low segment would eventually lead to a violation of the fuel economy constraint. Since this strategy is not feasible, the solution arising from our model needs to be adjusted to reflect the fuel economy constraint.

As Appendix VIII notes, this involves decreasing each product's marginal profit by some fraction of the amount by which the product's fuel-efficiency (measured in gallons per mile) falls short of the regulated amount.

TABLE IX:
CAFÉ Adjustment in # of ENTRIES

Segments	Pre-CAFÉ Net Profit ($)	How much gas used per mile exceeds CAFÉstandard (in gallons)	Shadow Price of CAFÉ ($)	Post CAFÉ Net Profit ($)	Post-CAFÉ R-Ratio	Number of Competitor Entries	Post CAFÉ # of Entries	Pre CAFÉ # of Entries	Excess Fuel Used (in gallons)
LOW	990	-62	19.6	2200	19	16.8	18	12	-2E+08
MID	3600	250	19.6	-1300	-13	17.7	0	25	0
LARGE	3700	450	19.6	-5100	-7	1.7	0	3	0
LUXURY	8800	450	19.6	5	0	18.3	1	23	5E+07
SPORT	7400	250	19.6	2500	5	11.4	7	13	2E+08
VANS	4500	0	0	4500	20	13.3	16	16	0
PICKUPS	9700	0	0	970	73	26.1	44	44	0
UTILITY	2400	0	0	2400	23	12.2	17	17	0
									-1E-05

To compute this shadow price, we initially specify input a starting estimate into our spreadsheet (corresponding to column 4 of Table IX) and compute the resulting revised net profit and number of entries. We then compute the fuel consumption associated with the portfolio (in the last column of Table IX). We then use Excel's Goal-Seeking Tool to specify the smallest value of the shadow price in column 4 which leads to zero (or negative) excess fuel consumption in the last column. This gives us our optimal portfolio which involves withdrawing from all car segments except for the low segment and the sports segment. The number of car entries is reduced from 65 to 25. Hence the CAFÉ regulation led to a dramatic reduction in the effective number of entries.

(6.2) The Effect of Improved Positioning

This analysis estimated the average development costs per effective entry using the firm's existing portfolio and its existing positioning of products. But improving the positioning of the firm's products might be an economical way of increasing the effective number of entries. As a result, improved positioning could lower the average development costs per effective entry and change our solution. To understand how optimal positioning might affect our solution, we now estimate how much optimal positioning might increase the effective number of entries.

To develop an ideal positioning, we rewrote our market share model (for each segment) as an anti-ideal point model by rewriting $u_i+S(T|i)$ as the weighted squared distance between product i's attributes and the segment anti-ideal point, $b^{\#}$. We then use principal component analysis to replace our attributes by factors and our matrix V^* by an orthogonal matrix of eigenvalues. Conducting such a principal component analysis using the data in Table II gives the following ten principal components:

TABLE X:
Principal Components

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
Reputation	1.08	0.02	0.12	0.18	0.18	0.19	0.00	0.08	0.17	0.03
Cargo	0.05	0.81	0.01	0.03	0.04	0.00	0.06	0.07	0.11	0.07
Quality	0.07	0.67	0.18	0.09	0.09	0.06	0.20	0.16	0.20	0.05
Workmanship	0.19	0.81	0.14	0.16	0.20	0.09	0.06	0.15	0.07	0.16
Features	0.02	0.74	0.01	0.07	0.10	0.10	0.07	0.20	0.01	0.05
Interior	0.00	0.99	0.02	0.05	0.16	0.14	0.16	0.14	0.02	0.03
Fuel	0.15	0.13	1.04	0.08	0.11	0.14	0.11	0.04	0.19	0.14
Acceleration	0.11	0.16	-0.54	0.05	0.04	0.17	0.02	0.01	0.15	0.15
Turning	0.13	-0.75	0.12	0.03	0.18	0.07	0.04	0.04	0.07	0.07
Towing	0.20	0.19	0.18	1.11	0.10	0.14	0.10	0.03	0.01	0.16
Eigenvalues	**97**	**63**	**48**	**18**	**3**	**1.1**	**0.8**	**0.4**	**0.08**	**0.03**

We dropped all but the most important factors. In the midsize segment, the eigenvalues associated with the first four factors accounted for about *97%* of the variance. Hence we dropped all but the four most important factors. (The first factor loaded heavily on brand reputation; the second factor on quality and interior roominess; the third factor on acceleration and fuel economy and the fourth factor on towing.) Finally we computed the anti-ideal point, $f^{\#}$, and the diagonal weighting matrix.

With these transformations, our market share model is in the form of an anti-ideal point model. Following DeSarbo & Rao(1986), we position each product as far from the anti-ideal point as possible on each dimension. To do so, we interviewed our engineers to assess feasible upper and lower limits on how much we could vary products on each of the factors. Because effectiveness is an exponential function of the squared difference between the factor score and $f^{\#}$, the products in an optimized portfolio will either have a rating of High on a given factor or a rating of Low:

TABLE XI:
A Maximally Differentiated Set of Products

Entry	Factor 1	Factor 2	Factor 3	Factor 4	Entry	Factor 1	Factor 2	Factor 3	Factor 4
1	High	High	High	High	**9**	Low	High	High	High
2	High	High	High	Low	**10**	Low	High	High	Low
3	High	High	Low	High	**11**	Low	High	Low	High
4	High	High	Low	Low	**12**	Low	High	Low	Low
5	High	Low	High	High	**13**	Low	Low	High	High
6	High	Low	High	Low	**14**	Low	Low	High	Low
7	High	Low	Low	High	**15**	Low	Low	Low	High
8	High	Low	Low	Low	**16**	Low	Low	Low	Low

The first combination corresponds to a prestige brand with considerable family functionality, towing and sportiness. The last combination corresponds to an economical brand with basic performance on all attributes.

We now compute the effectiveness of each of these products and order all the entries in order of effectiveness. Since the actual number of effective entries for the midsize segment was *7.5*, we built a portfolio with an effective number of entries of *7.5* by choosing the entries of highest effectiveness. We computed the corresponding average development cost and found it *30%* lower than the development cost associated with the existing portfolio of mid sized products. (Since the optimal number of effective entries associated with this lower development cost was higher than *7.5*, simultaneously optimizing the number of entries and their positioning might have led to somewhat different conclusions.) We repeated this procedure for each of the other segments. Unfortunately the average reduction in development costs was much lower for other segments so that the overall reduction in average development costs across segments was only *8.5%.* As a result, improving product positioning would not have had as big an impact on profit as, for example, reducing marginal costs.

Hence these sensitivity analyses suggest that management's main problem was not how to optimize the effective number of entries; the main problem was how to optimize the quality, cost and fuel-efficiency of those entries.

7. CONCLUSIONS

There's often considerable arbitrariness both in how a product's performance on an attribute is measured and in how firms count the number of distinct entries they have. This is especially evident in the automobile industry where the number of product entries can be estimated to be less than a hundred (if vehicles built off the same platform are treated as identical), several hundred (if every distinct make is considered a product line), almost a thousand (if every distinct nameplate and trim level is considered a product line), more than three thousand (if each distinct nameplate, trim, engine & seating combination is called a product line.)

To eliminate this arbitrariness, this paper introduced our own procedures for quantifying performance on an attribute and for counting the number of distinct entries. These procedures substantially simplified how market demand depended on product attributes and on how profit depended upon the number of entries. As a result, it allowed us to construct a very simple model of firm profit.

This profit model has many implications for the management of the firm, e.g.,

1. In positioning its portfolio, a firm should balance the value of having products which score well on the dimensions which are, on average, the most important dimensions against the need for diversity in the product portfolio. When individual preferences are extremely heterogeneous, diversity becomes much more important than scoring well on the dimensions that are, on average, most important.
2. Development costs should be measured in terms of development cost per effective entry. A firm which cuts its investment spending by *25%* in order to produce a product that is only equivalent to *50%* of its previous entry has, in effect, increased its development costs by *50%.* If it's cheaper for a firm to develop two *50%* effective entries than a single *100%* effective entry, then the firm should develop two *50%* effective entries. Otherwise the firm should develop a single *100%* effective entry.
3. The firm's market share is heavily determined by a single statistic, the ratio of its profit margin to its development costs. Firms with low profit margins and low development costs and firms with high profit margins and high development costs could both have comparable market shares. Hence two dramatically different strategies lead to comparable market shares.

4. The time between when a product achieves its peak sales and the time when the firm makes its peak investment in product development has a major impact on the profitability of the product line and on the firm's equilibrium market share.
5. The firm's overall profit is determined not by the ratio of profit margin to development costs but by the difference of their square roots. As a result, a firm with high profit margins and high development costs will still tend to have higher overall profits than the firm with low profit margins and low development costs

In practice, the value of this kind of model lies in directing executive attention to the major factors driving firm value (i.e., *in ensuring that executives are having the right kinds of conversations.)* Since overall profit was not extremely sensitive to the effective number of entries in the automotive example, our most important practical finding was that executive attention should be focused, not on adjusting the number of product entries, but on improving their quality and cost.

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APPENDIX I:

Suppose there are M products with m attributes in a market of N customers. For each product i, let $b_k(i)$ be product i's rating on attribute k and let $b(i)$ be the column vector $(b_1.(i),\ldots.b_m(i))^T$. Let w be the importance weights attached to each of the m attributes. Let p_i be the probability of buying product i. Assumptions 1,2,3 and 4 imply

$$p_i = \sum_w f(w) \frac{exp(b(i)\, w^T)}{[\sum_b \; exp(b\, w^T)]\,\}}$$

where $f(w)$ is a normal probability densities. Now $\sum_b \ exp(b\, w^T)]/M$ is the average value of $exp(bw^T)$. Let $g(b)$ be a normal probability distribution with mean Eb and variance-covariance C. Then we can also compute the average value of $exp(bw^T)$ by integrating $exp(bw^T)$ over $g(b)$. In other words,

$$\sum_b \ exp(b\, w^T)]/M \ = \ \int exp(bw^T)\, g(w)\, dw$$

But the value of this integral is just $exp(E(b)w^T + wCw^T/2)$. Hence

$$[\sum_b \ exp(b\, w^T)] \} = N\, exp(E(b)w^T + wCw^T/2)$$

As a result, the choice probability becomes

$$p_i = \sum_w f(w)\, exp([b(i)\text{-}Eb]w^T \ - wC\, w^T/2)$$

If $f(w)$ is a normal density with

$$f(w) = (2\pi|V|)^{-1/2} \ exp(-(w\text{-}Ew)^T\, V^{-1}(w\text{-}Ew)/2)$$

then p_i is proportional to

$$\int exp\{-(w\text{-}Ew)^T\, (V^{-1}/2)(w\text{-}Ew)+[b(i)\text{-}Eb]w^T \ - w(C/2)\, w^T \ \}\, dw$$

Letting $(V^*)^{-1} = (V^{-1}+C)$ and $b^*=b(i)\text{-}Eb+EwV^{-1}$ implies that p_i is proportional to

$$\int exp(b^*w^T - w\, (V^*)^{-1}\, w^T /2)\, dw$$

where we ignore all terms that don't depend upon b* or w. This is proportional to

$$\int exp(-(w\text{-}b^*V^*)((V^*)^{-1})(V^*b^*\text{-}w)/2)exp(+b^*(V^*)^{-1}b^*/2)\, dw$$

Integrating out w and dropping terms that don't depend on b^* gives p_i proportional to

$$exp(b^*V^*\, b^*/2)$$

If we define $b^\# = Eb\text{-}EwV^{-1}$ and interpret $b^\#$ as an anti-ideal point, then this is in the form of an anti-ideal point model where the distance measure is an exponential function of the quadratic difference between attribute scores.

Since $b^* =b(i)\text{-}Eb+EwV^{-1}$, substituting and dropping terms that don't depend on $b(i)$ gives p_i proportional to

$$exp(b_i(V^*)^{-1}\, V^{-1}Ew +\{(b_i\text{-}Eb)V^*(b_i\text{-}Eb)/2)$$

Setting $w^* = (V^*)^{-1}\, V^{-1}\, Ew$ proves the result.

APPENDIX II

Define

$$L_i(w) = \Sigma\, w_k\, B_{jk}\; - ln(\Sigma_j\, exp(\Sigma\, w_k\, B_{jk}))$$

so that Assumptions (1),(2) and (3) imply

$$p_I = \int f(w)\; exp(L_I(w))\; dw$$

A simple Taylor Series approximation about $w=(w_1....w_n)=(0,....0)$ gives

$$L_i(w) = L_i(0) + \Sigma_k\, w_k\, [dL/dw_k\,] + (1/2)\; \Sigma\Sigma_{kr}\; w_k w_r\; d^2L/dw_k dw_r]$$

We get

$$dL/dw_k = b_k(i) - \Sigma_j \frac{b_k(i)\; exp(\Sigma w_k b_k(i))}{[\Sigma_j\, exp(\Sigma w_k b_k(j)))]} = b_k(i) - \Sigma b_k(j)/T\; = b_k(i) - Eb_k$$

$$d^2L/dw_k dw_r = -\; \Sigma_j \frac{b_k(i) b_r(i)\; exp(\Sigma w_k b_k(i))}{[\Sigma\, exp(\Sigma w_k\, b_k(i)))]} +$$

$$\Sigma \frac{b_k(i)\; exp(\Sigma w_k b_k(i))}{[\Sigma exp(\Sigma w_k\, b_k(j))]}\;\; \Sigma \frac{b_r(j) exp(\Sigma w_k\, b_k(j))}{[\Sigma exp(\Sigma w_k\, b_k(j)))]}$$

$$= -\; \Sigma_i\; (b_k(i) b_r(i))/T + Eb_k Eb_r$$

If C_{kr} is the *physical covariance* between attributes k and r, then

$$d^2L_i/dw_k dw_r = -C_{kr}.$$

Hence

$$p_i = \int f(w)\;\; exp(L) = \int f(w)\;\; exp(L(0) + (b_i - Eb)w - (1/2)\; w^T\, Cw)$$

The remainder of the proof is just a repetition of the second part of the proof of Proposition 1.

APPENDIX III

Let P be marginal profit. Let S be the total sales volume of a product over its lifetime and marginal profit. Suppose profit at a particular time is $SP\, f(t)$. If r is the rate of interest, then the total discounted profit is

$$\int SP\, f(t)\; exp(-rt)$$

If $f(t)$ is a gamma distribution, then this becomes

$$S\,P\ \int exp(-rt)exp(-kt)(t)^a / \int exp(-kt)(t)^a$$

If $s=t(k+r)/k$, then this can be rewritten as

$$SP\ \int exp(-ks)\ (ks/(k+r))^a / \int exp(-kt)t^a = SP/(1+(r/k))^a$$

For the gamma distribution, the expected time is a/k. If we let ET be the expected time at which a vehicle is introduced, then we can write this as $SP/[1+ ET/a]^a$ which is approximately $SP/(1+r)^{ET}$.

APPENDIX IV

Letting E(.) denote the expectation operator and letting capacity be c implies that the expected profit associated with having capacity c given product demand D and per unit capacity cost,C, is

$$P\ E[Min(D,c)] - Cc$$

Thus the optimal value of capacity,c, satisfies $Pr(c=D) = C/P$. If D is randomly distributed with a centering parameter, ED, and a scaling parameter, $s\ ED$, then we can write

$$D = ED + z\ (s\ ED)$$

where z is a standardized random variable. We can similarly write capacity as

$$c = ED + c^*\ (s\ ED)$$

Making these substitutions gives an expected profit of

$$ED(P\text{-}C)\ +\ (s\ ED)\ \{ P[E[Min(z,c^*)]\text{-}\ C\ c^*]$$

Since the optimal capacity satisfies $Pr(c=d)=C/P$, we also have $Pr(c^*=z)=C/P$, If $G(z)$ is the probability density of z and G^{-1} is the inverse of that density, then $c^*=G^{-1}(C/P)$ so that, if G is normal, $c^*= [2ln(P/C)]^{1/2}$. We define s_F , the safety factor, as

$$s_F = -\{P\ E[Min(z,G^{-1}(C/P))]\text{-}C\ G^{-1}(C/P)\}/C$$

so that profit per unit of expected demand equal to $(P\text{-}C - s\ s_F C)$. For our automotive demand, s_F was relatively constant across all automotive segments.

APPENDIX V:

Optimizing $SP\, d\, n_a/[n_a+n^*_a]-Cn_a$ implies that

$$SPd\, n_a /[n_a+n^*_a]^2 = K$$

(The solution of this equation is an optimum since the second derivative of this term is negative.). Hence we need $[n_a+n^*_a]^2=[SPd\, n_a/K]$. Letting $R=[SPd/C]$ gives

$$[\Sigma n_r]=n_a+n^*_a=[Rn^*_a]^{1/2}$$

We then compute

$$n_a = (n_a+n^*_a)-n^*_a=(Rn^*_a)^{1/2} - n^*_a .$$

To compute overall profit, substitute $(n_a+n^*_a) = (Rn_a)^{1/2}$ into $SPd\, n_a/(n_a+n^*_a) - K^*n_a$ to get

$$SPd\, n_a/[n_a+n^*_a]-K\, n_a =S\, Pd - Cn_a - P[n^*_a /[n_a+n^*_a]]=$$
$$SPd-Kn_a-K[n_a+n^*_a]=SPd-C[2n_a+n^*_a] =SPd-C[2(Rn^*_a)^{1/2}-n^*_a] =$$
$$SPd+Kn^*_a-2(PK\, n^*_a)^{1/2}= \{[SPd]^{1/2} - [Kn^*_a]^{1/2} \}^2$$

APPENDIX VI:

The number of competitive entries,n, arises from the various firms *1….F* as well as from the zero option of not buying any product. We now let n_r denote firm r's effective number of entrants and n_r denote the effective number of competitors to firm i. Let R_i denote firm's i profit to investment ratio. Then

$$n = (n_i+n^*_i)= (R_i(n-n_i))^{1/2}$$

is the same for all firms $i=1…F$. Hence for any firm a

$$R_i(n-n_i)=R_a(n-n_a)$$

and

$$n_i = n + (R_a/R_i)(n_a-n)$$

If we let F be the total number of firms, then summing over all firms i gives

$$n = F\, n + R_a\, (n_a-n)\, \Sigma(1/R_i)$$

Since $n=[R_a(n-f_a)]^{1/2}$, this becomes

$$n = Fn - n^2\, \Sigma(1/R_i)$$

Suppose we define R by

$$(1/R)=(1/F)\ \Sigma(1/R_i)$$

as a harmonic average of the R-factors of different firms. Then

$$n = Fn - n^2 F/R$$

so that

$$n = R(F-1)/F$$

Since $n=(R_a(n-n_a))^{1/2}$, we have

$$n^2=R_a(n-n_a)$$

or

$$(n_a/n) = 1-n/R_a = 1 - (R/R_a)(F-1)/F = \{F+(1-F)(R/R_a)\}/F$$

APPENDIX VII

Suppose we assume, instead, that P is inversely proportional to market size, specifically

$$P = P_0/(f+n)^h$$

for some positive constant, h. We can again solve for the optimal portfolio size, f. In this case, our formula for overall profit becomes

$$SP_0\ df/(f+n)^{h+1} - C f$$

For this model, the breakeven point has changed from (SdP_0/C) to the smaller value of $(SdP_0/C)^{1/(h+1)}$. Likewise the optimal portfolio size will be lower (when h>0). (This effect is minimal when the size of the competitor portfolios, n, is large.)

APPENDIX VIII

Suppose the firm sells x_i products in segment i which have a fuel economy(in miles per gallon) of F_i . Then the corporate average fuel economy constraint requires that

$$\Sigma x_i/[\Sigma (x_i/F_i)] > F$$

where F is the mandated fuel economy target. This can be written more simply as

$$\Sigma (x_i/F_i) > (1/F)(\Sigma x_i) \text{ or as } \Sigma x_i[(1/F_i)-(1/F)]]>0$$

Let $G_i =(1/F_i)-(1/F)$ be the amount by which the actual number of gallons required to travel a mile exceeds the amount implied by the standard.. If the effective number of entries in segment I is n_i, then the firm's sales in segment i are $S\ n_i/(n_i+n^*_i)$. If P_i denotes the marginal profit of product i, then the profit-maximizing firm maximizes

$$\sum S\, P_i\text{-}n_i/(n_i+n^*_i) - K\, n_i \quad \text{subject to this constraint.}$$

Introducing the Lagrangian multiplier q allows us to write this as

$$\sum SP_i n_i/(n_i+n^*_i) - K_i n_i \ + \ q \ \sum_i G_i\, S\, n_i/(n_i+n^*_i)$$

or

$$\sum S[P_i + q\, G_i]\, n_i/(n_i+n^*_i) - K_i\, n_i$$

where q must be chosen to ensure that the constraint holds. Solving as before gives

$$n_i + n^*_i = \{(S[P_i+qG_i\,])/K_i\,\}^{1/2}$$

where q must be chosen to ensure that

$$\sum_i S\, G_i\, n_i\, /[n_i+n^*_i\,] > 0$$

Substituting for $n_i+n^*_i$ gives

$$\sum_i G_{\,i} n^*_i \ \ [SK_i/(P_i+qG_i)]^{1/2} < \sum_i SG_i$$

We can solve this expression iteratively to find q.