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Multiattribute Preference Analysis with Performance Targets

Robert F. Bordley
General Motors, Pontiac Centerpoint North Campus, MC 483-585-372, 585 South Boulevard, Pontiac, Michigan 48341, robert.bordley@gm.com

Craig W. Kirkwood
Department of Supply Chain Management, W. P. Carey School of Business, Arizona State University, Tempe, Arizona 85287-4706, craig.kirkwood@asu.edu

This paper develops an approach based on performance targets to assess a preference function for a multiobjective decision under uncertainty. This approach yields preference functions that are strategically equivalent to conventional multiattribute utility functions, but the target-oriented approach is more natural for some classes of decisions. In some situations, the target-oriented preference conditions are analogous to reliability theory conditions for series or parallel failure modes in a system. In such cases, reinterpreting the conditions using reliability concepts can be useful in assessing the preference function. The target-oriented approach is also a generalization of common forms of goal programming. The approach has particular applicability for resource allocation decisions where the outcome of the decision is significantly determined by the actions of other stakeholders to the decision, such as new product development or decision making in a controversial regulated environment.

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1. Introduction

This paper develops an approach based on performance targets to assess a preference function for a multiobjective decision under uncertainty. This approach is shown to yield preference functions that are strategically equivalent to conventional multiattribute utility functions, but the target-oriented approach is more natural for some classes of decisions. Therefore, this approach provides new methods to assess a preference function for use in certain multiobjective decision analyses.

The target-oriented approach is particularly applicable for resource allocation decisions where multiple stakeholders to the decision impact the success that results from the selected allocation of resources. Examples of such decisions include (1) budget allocation for new product development where competitors are simultaneously developing competitive products, and (2) allocation of project funding related to controversial activities in regulated environments. For example, a customer's purchase decision for a new product may involve comparing your product's performance against competing products on such attributes as cost, quality, and features. Your budget allocation decision for new product development in such a situation should consider two types of uncertainty: (1) the performance of your new product with respect to the attributes, and (2) the performance of competitors' products on these same attributes. The second type of uncertainty can be conceptualized by saying that your competitors' product performance establishes *targets* that your product will be compared against when a purchase decision is made. If the performance of your competitors' future products is uncertain when you make your product development budgeting decisions, the performance of your potential products must be compared against *uncertain* targets set by your competitors' products on the various attributes.

2. Background

Substantial empirical evidence indicates that the conventional concave single-attribute utility function often does not provide a good description of individual preferences. As a substitute, Kahneman and Tversky (1979) propose an S-shaped value function, and Heath et al. (1999) suggest that the inflection point in this S-shaped value function can be interpreted as a target. Developing this concept further, Castagnoli and Li Calzi (1996) present a target-oriented decision-making approach for decisions under uncertainty with a single evaluation attribute, and this type of decision-making approach is now discussed.

2.1. Normative Target-Oriented Formulation

For notational convenience, designate an evaluation attribute by Z, and an arbitrary specific level of that evaluation attribute by z. An expected utility decision maker is defined to be *target oriented* for a single-attribute decision if the decision-maker's utility for an outcome depends only on whether a target is achieved with respect to Z. Thus, a target-oriented decision maker has only two different utility levels, and because a utility function is only specified to within a positive affine transformation, these two utility levels can be set to one (if the target is achieved) and zero (if the target is not achieved). With this scaling for the utility function, a target-oriented decision-maker's expected utility for alternative a is

$$\begin{aligned} \mathrm{E}[u \mid a] &= \int_{z=-\infty}^{\infty} \big\{p(z \mid a) \times 1 + [1 - p(z \mid a)] \times 0\big\} f(z \mid a)\, dz \\ &= \int_{z=-\infty}^{\infty} p(z \mid a) f(z \mid a)\, dz, \end{aligned}$$

where $p(z \mid a)$ is the probability that the target is achieved given that the attribute is at level z and alternative a is selected, and $f(z \mid a)$ is the probability density function for z given that a is selected.

If targets are probabilistically independent of alternatives, *once z is specified*, this reduces to

$$\mathrm{E}[u \mid a] = \int_{z=-\infty}^{\infty} p(z) f(z \mid a)\, dz, \qquad (1)$$

where $p(z)$ is the probability that the target is achieved given that the attribute is at level z. Thus, for a target-oriented decision maker it is not necessary to assess a utility function; instead, it is necessary to determine the probability function $p(z)$. As we discuss below, in some decision contexts this may be a more intuitively appealing task than assessing a utility function. A special case of (1) is where the target is known for certain, and hence $p(z)$ is either zero or one, depending on z. Therefore, the target-oriented approach applies to decisions with certain targets as well as decisions with uncertain targets.

It is clear from (1) that there is always an equivalent standard (nontarget-oriented) utility formulation for any target-oriented formulation because the utility function can be set equal to $p(z)$ to create such a formulation. The converse is also true: Because utility functions consistent with the Savage axioms are bounded (Fishburn 1970), the utility function can be rescaled so that it lies between zero and one over the range of levels for Z that is of interest. Then, setting $p(z)$ equal to the (possibly rescaled) utility function creates a strategically equivalent target-oriented formulation.

2.2. Descriptive Target-Oriented Formulation

The decision making described in the preceding section is *normative* in the sense that it assumes the decision maker wishes to obey the axioms of rational choice (Von Neumann and Morgenstern 1947, Pratt et al. 1964) that yield expected utility as the decision criterion. An alternative formulation that also leads to (1) uses a target-oriented formulation to model *descriptively* the behavior of a decision maker. This approach models a decision maker as someone who intuitively ranks alternatives in accordance with their probability of achieving a possibly uncertain target on Z, and hence selects the alternative with the greatest probability of achieving this target. Such a model might be considered a natural variant on Simon's theory of bounded rationality. With this approach, the right-hand side of (1) is interpreted as the probability that the target will be achieved, given that alternative a is selected. Of course, there is always a normative expected utility interpretation of (1) that is mathematically equivalent to this descriptive interpretation.

3. Target-Oriented Multiattribute Decisions

Here are several illustrative decision-making situations where a target-oriented multiattribute decision-making approach is natural.

Product Development. A target-oriented approach to product development resource allocation is natural in some situations where the product is a complex system, for example, an automobile, a personal computer, a television, a new missile defense system, or any of a variety of other complex products that are developed or improved by companies in competitive markets. Typically such products are developed by sizable teams, with a key decision being the resources (budget, personnel, office space, testing capacity, etc.) that will be allocated to the subteams working on each subsystem of the product. Most such products are developed in a competitive environment against other companies (or other opponents such as foreign military powers), and the success of the new or improved product will be determined by how well it performs relative to the competing products. Because competing products are usually under development at the same time, they give rise to uncertain performance targets against which your product will be compared when it is finally released. A target-oriented decision-making approach is relevant if potential customers for the product make purchase decisions based on whether your product outperforms the competition on various sets of attributes. Examples of such decision rules include:

1. *The Pugh Rule.* This widely used concept selection process (Pugh 1991) starts with a benchmark concept and compares each proposed concept against the benchmark on several criteria. The proposed concept that is superior to the benchmark on the largest number of criteria is chosen for further development. The Pugh rule is commonly implemented along with Quality Function Deployment (Clausing 1998).

2. *Plurality Voting.* The customer chooses the product that is superior to other products on the greatest number of criteria.

3. *Elimination by Aspects.* The customer ranks attributes in order of importance, and starting with the most important attribute eliminates products that are inferior on that attribute, repeats this procedure with the next most important attribute, and continues in this way successively eliminating products that are inferior on successively less important attributes (Tversky 1972).

On the other hand, if potential customers make decisions based on weighted average performance across attributes, product development teams will work to develop a product whose weighted average performance exceeds the weighted average performance of competing products. In this case, a target-oriented approach with a single attribute (namely, weighted average performance across the performance attributes) is appropriate.

Regulated Environments. Much business decision making is conducted in an environment subject to government regulation or standards with substantial scrutiny by interested parties. Examples of this include constructing new facilities such as warehouses, stores, manufacturing plants, pipelines, or power plants; or conducting certain types of business, such as the use of genetically modified plants or animals; or manufacturing involving or producing hazardous materials. In such decision-making environments, the performance required with respect to various performance attributes by the different stakeholders can be uncertain, and hence decision making is done in the presence of uncertain targets.

Often each stakeholder group in such a situation focuses almost exclusively on performance with respect to the evaluation attribute of interest to them. Hence, failure to provide adequate performance with respect to each attribute, or at least with respect to a sufficient number of attributes, will lead to failure of the selected alternative. In such situations, a target-oriented decision-making approach is appropriate.

Setting Performance Standards. It is common in operations management to set performance targets, such as monthly sales quotas for sales personnel; or quality, throughput, and safety standards for manufacturing plants. These are often set to provide concrete incentives for operations personnel who may have difficulty relating their daily personal activities to higher-level, somewhat abstract, corporate targets involving market share, revenue growth, net income, etc. These higher-level corporate targets can be uncertain because these in turn address the highest objective of a business, which is usually to be profitable and stay in business. Thus, the setting of performance standards is often decision making with uncertain targets, and hence a target-oriented approach is appropriate.

Resource Allocation Under Uncertain Competition. Product development, regulated decision making, and setting performance standards are examples of decisions with three characteristics: (1) a fixed resource must be allocated among competing uses to produce an uncertain final result, (2) there are (possibly uncertain) targets with respect to multiple performance attributes, and (3) the final success of the decision outcome is measured by the extent to which the multiple performance targets are met, and not by detailed performance with respect to each performance attribute. Whenever these characteristics are present, a target-oriented approach to multiattribute preference analysis is appropriate.

The examples above emphasize that performance targets may be uncertain, but decisions where some or all of the targets are known for certain are special cases that are also covered by the results developed below.

4. Target-Oriented Multiattribute Formulation

This section extends the target-oriented approach in §2 to decisions with multiple evaluation attributes. We first consider a normative formulation that is analogous to the normative formulation presented in §2.1 for the single-attribute situation.

Definition 1. With n attributes $X = (X_1, X_2, \ldots, X_n)$, a decision maker is defined to be *target oriented* if his or her utility for an outcome $x = (x_1, x_2, \ldots, x_n)$ depends only on which targets are met by that outcome, where there is a single target for each attribute.

For example, a decision maker allocating a product development budget to upgrade an existing product might have three evaluation attributes: cost, quality, and features. If the decision maker is target oriented, alternative budget allocations will be ranked based only on which of the cost, quality, and features targets are met, and not on the specific levels that are achieved for the three attributes. These performance targets might be set relative to a competitor's project that is also currently under development, and hence might be uncertain due to the uncertain performance of the competitor's product.

It follows from Definition 1 that the utility function for a target-oriented decision maker is completely specified by $2^n - 2$ constants where these constants are the utilities of achieving specific combinations of the various targets. (There are 2^n such constants, but because a utility function is only specified to within a positive affine transformation, two of these can be specified arbitrarily.) Therefore, to calculate expected utilities it is necessary to know the probability for each of the 2^n different possible combinations of target achievement as a function of the levels for the n attributes. Define $I = (I_1, I_2, \ldots, I_n)$ as a set of indicator variables where I_i equals one if the target for X_i is achieved and zero otherwise. Let I_u be the set of all 2^n combinations of possible levels of I, and let $p(I \mid x)$ be the probability of I given x. Then, the expected utility for alternative a is

$$\mathrm{E}[u \mid a] = \int_X \left[\sum_{I \in I_u} u(I) p(I \mid x) \right] f(x \mid a)\, dx, \tag{2}$$

where $f(x \mid a)$ is the probability density function over X given a, and $u(I)$ is the decision maker's utility function over I.

DEFINITION 2. For notational convenience, define the *target-oriented preference function* $u_T(x)$ by

$$u_T(x) \equiv \sum_{I\in I_u} u(I)p(I \mid x). \tag{3}$$

Using this definition, (2) shows that a target-oriented decision-maker's expected utility equals the expected value of his or her target-oriented preference function. Equation (3) can be rewritten as

$$u_T(x) = \sum_{I\in I_u} k_I p(I \mid x), \tag{4}$$

where $k_I = u(I)$, and hence the decision-maker's utility function is specified by a set of 2^n constants, where these are the utilities of achieving each unique combination of targets.

Note that there is a descriptive formulation that is equivalent to (2), just as there is a descriptive formulation for the single-attribute case that is equivalent to (1). Specifically, if $u(I)$ is interpreted as the probability that a particular set of target achievements I is "good enough" with respect to the entire set of targets, then the right side of (2) gives the probability that a particular alternative will be "good enough" for the decision maker to select this alternative. Hence, if it is assumed that a decision maker will select the alternative that has the greatest probability of being "good enough," then (2) provides an approach to descriptively model decision-making behavior in multiobjective decisions.

5. Special Cases

5.1. Independent Targets

We now examine conditions on a decision-maker's preferences that lead to specific functional forms of (4). These conditions are useful because they can simplify the assessment of the target-oriented preference function in practical applications.

DEFINITION 3. If a decision-maker's probability of achieving the target on any attribute X_i depends only on the level x_i, then the decision maker is said to have *independent targets*.

Independent targets seem intuitively reasonable for many target-oriented decisions, and with independent targets, the I_i are probabilistically independent, given x. If we define $p_i(x_i)$ to be the probability that the target for attribute X_i is achieved, given the level x_i of that attribute, then independent targets imply

$$p(I \mid x) = \left\{ \prod_{\substack{i=1\\ i\ni(I_i=1)}}^{n} p_i(x_i) \right\} \left\{ \prod_{\substack{i=1\\ i\ni(I_i=0)}}^{n} [1-p_i(x_i)] \right\}, \tag{5}$$

where a product is defined to be one if there are no factors. Equation (5) combines with (4) to yield

$$u_T(x) = \sum_{I\in I_u} k_I \left\{ \prod_{\substack{i=1\\ i\ni(I_i=1)}}^{n} p_i(x_i) \right\} \left\{ \prod_{\substack{i=1\\ i\ni(I_i=0)}}^{n} [1-p_i(x_i)] \right\}. \tag{6}$$

For example, with two attributes this becomes

$$\begin{aligned} u_T(x_1,x_2) = {} & k_\phi[1-p_1(x_1)][1-p_2(x_2)] \\ & + k_1 p_1(x_1)[1-p_2(x_2)] + k_2[1-p_1(x_1)]p_2(x_2) \\ & + k_{12} p_1(x_1) p_2(x_2), \end{aligned} \tag{7}$$

where $k_\phi = u(0,0)$, $k_1 = u(1,0)$, $k_2 = u(0,1)$, and $k_{12} = u(1,1)$.

Note that $u_T(x)$ in (6) is fully determined by the n functions $p_i(x_i)$ and the 2^n constants k_I. Thus, the amount of information needed to determine $u_T(x)$ when there are independent targets is the same as that needed to determine a multilinear multiattribute utility function (Keeney and Raiffa 1976, §6.4; Kirkwood 1997, Theorem 9.41). In fact, Theorem 2 below shows that it is always possible to find a multilinear utility function that is *strategically equivalent* to $u_T(x)$ in (6), where two preference functions for decisions under uncertainty are said to be strategically equivalent if they give the same rank ordering for any set of alternatives and hence are positive affine transformations of each other (Keeney and Raiffa 1976, Theorem 4.1; Kirkwood 1997, Theorem 9.25).

5.2. Additive Target Preferences

Following standard terminology, a decision maker is said to have *additive independent preferences* if the decision-maker's rank ordering for any set of alternatives depends only on the marginal probability distributions over the attributes for each alternative (Keeney and Raiffa 1976, §6.5; Kirkwood 1997, Definition 9.31).

THEOREM 1. *The target-oriented preference function for a target-oriented decision maker with independent targets and additive independent preferences is strategically equivalent to*

$$u_T(x) = \sum_{i=1}^{n} K_i p_i(x_i) \tag{8}$$

for some constants K_i. If an alternative that achieves all the targets of a second alternative and also achieves additional targets is not less preferred than the second alternative, then $K_i \geqslant 0$ for all i.

The proofs for this theorem and the others below are in the appendix. The form of u_T in (8) is similar to the form for the standard additive utility function, and Theorem 3 below shows that it is always possible to determine an additive utility function that is strategically equivalent to an additive u_T.

5.3. Reliability-Structured Target Preferences

While the additive target-oriented preference function shown above has a similar form to the standard additive multiattribute utility function, we now consider a type of target-oriented preference structure that is different from the structures usually assumed for multiattribute utility functions.

DEFINITION 4. A target-oriented decision maker with independent targets is said to have a *reliability target structure* if $u(I)$ can take on only two different levels. An outcome with the higher utility level is called a *success*, and an outcome with the lower utility level is called a *failure*.

Because a utility function is only defined to within a positive affine transformation, it is always possible with a reliability target structure to scale $u(I)$ so that the utility for a success is one and the utility for a failure is zero. This scaling convention is used for the remainder of this paper.

The potential usefulness of a reliability target structure is illustrated by the new product development example discussed above where there are three evaluation attributes: cost, quality, and features. One way to analyze this decision is as follows: Suppose that potential customers for this new product will compare it against the competitors' similar products, and hence a target-oriented decision analysis approach may be appropriate. Because these competing products are not yet on the market, the targets on the three attributes that the new product must achieve to be a success are uncertain. A reliability target structure may be appropriate if the decision is viewed as selecting the alternative with the highest likelihood of acceptance by potential customers. For example, a potential customer may purchase our product only if it is of higher quality than the competitors' products and is also either less costly or has more features. Assuming that the targets for the attributes are independent and $u(I)$ is scaled to be either zero or one as discussed above, then

$$u_T(x_c, x_q, x_f) = p_q(x_q)\{1 - [1 - p_c(x_c)][1 - p_f(x_f)]\},$$

where the subscripts c, q, and f refer to cost, quality, and features, respectively.

This is a special case of (6), but the k_I do not need to be directly assessed because they are determined by specifying the combinations of targets needed for success. This shows the origin of the term *reliability target structure*, because $u_T(x)$ is determined using analogies with parallel and series elements in reliability. (The cost and features targets are analogous to parallel elements in reliability analysis, and the quality target is analogous to an element in series with these.) Theorem 4 below shows that there is an equivalent multiplicative utility function for many reliability-structured target-oriented preference functions.

6. Assessment Procedure with an Application

6.1. Assessment Procedure

Assessing a target-oriented preference function of the form of (6) requires determining the n target probability functions $p_i(x_i)$ and the 2^n constants k_I. While there are no theoretical restrictions on the shape of the $p_i(x_i)$, in many applications these will be monotonic in x_i with greater levels of x_i either always having a higher probability or always having a lower probability. For example, in the new product development example discussed above, it seems reasonable that $p_c(x_c)$ would monotonically decrease with increases in x_c. That is, lower costs will always lead to a higher probability of meeting the cost target.

In a decision with monotonic $p_i(x_i)$, these functions might be determined by assessing the probability distribution for the minimum level of X_i that will meet the target. Thus, for the new product development example, probability distributions might be assessed for the level of performance that the competitors will achieve in their products with respect to cost, quality, and features. Then, either the cumulative distribution or the complementary cumulative distribution for this uncertain quantity would be used as $p_i(x_i)$, depending on whether more or less of an attribute is preferred.

Assessment of the k_I could be time consuming because there are 2^n of these. However, in analogy to applications of multiattribute utility theory (Corner and Kirkwood 1991), we might assume that additive independence holds so that only the n constants in (8) must be assessed. For example, some variation on the SMART assessment procedure (Edwards 1977, Edwards and Barron 1994) might be appropriate to assess these constants. This assessment is simplified even further if a reliability target structure is assumed. Then, it is only necessary to determine the appropriate series and parallel target structure, as illustrated by the new product development example discussed above. In this case, no weights have to be determined, but instead the parallel/series structure is specified.

6.2. Illustrative Application—New Product Development

Keeney and Lilien (1987) consider a decision where a company wanted to assess how prospective customers would evaluate a proposed new tester for very large-scale integrated circuits. They identified four categories of evaluation criteria (technical, economic, software, and vendor support) with a total of 17 evaluation attributes, as shown in the first column of Table 1. The preference monotonicity for each evaluation attribute is shown in the second column of Table 1, and the scores on each of the evaluation attributes are shown in the third through fifth columns of the table for the OR 9000, which was the proposed new tester, and its two competitors, the J941 and the Sentry 50.

Table 1. New product application—data.

Evaluation attribute	Monotonicity	Tester ratings		
		OR 9000	J941	Sentry 50
Technical				
Pin capacity	Increasing	160	96	256
Vector depth	Increasing	0.128	0.256	0.064
Data rate	Increasing	50	20	50
Timing accuracy	Decreasing	1,000	1,000	600
Pin capacitance	Decreasing	55	50	40
Programmable measurement units	Increasing	8	2	4
Economic				
Price	Decreasing	1.4	1	2.8
Uptime	Increasing	98	95	95
Delivery time	Decreasing	3	6	6
Software				
Software translator	Increasing	90	90	90
Networking: Communications	Increasing	1	1	1
Networking: Open	Increasing	1	0	0
Development time	Decreasing	3	4	4
Data analysis software	Increasing	1	1	1
Vendor support				
Vendor service	Decreasing	2	4.75	6
Vendor performance	Decreasing	4	4	4
Customer applications	Increasing	1	1	1

Keeney and Lilien (1987) assessed the measurable value function for a lead user at a primary customer company for this testing equipment. This lead user first assessed a minimum acceptability level and a maximum desirability level for each attribute. Keeney and Lilien then confirmed that the user's preferences were describable by an additive measurable value function, and they assessed a single-dimensional value function and an importance weight for each attribute. The assessed additive measurable value function was then used to evaluate the OR 9000 against the J941 and Sentry 50, and the results served as input to determine that the proposed new tester was not competitive enough to market.

For this decision, it is natural to think in terms of performance targets because the explicit purpose of the analysis was to determine whether the OR 9000 was attractive when judged against the J941 and the Sentry 50. Thus, the performance of these two testers sets targets against which the OR 9000 is judged. Table 2 illustrates a possible target-oriented preference analysis for this decision. There is no uncertainty about the performance of the three testers, and preferences are monotonic with respect to each evaluation attribute. Therefore, the target will be achieved for an evaluation attribute only if performance meets or exceeds a target performance level on that evaluation attribute. The target performance levels might be set using different criteria for each evaluation attribute. For example, there might be some evaluation attributes where it would be judged necessary only to meet some minimal level of performance, while for other evaluation attributes it might be judged necessary to exceed the performance of both of the competitors by some threshold amount (for example, 10%).

To illustrate a possible target-oriented analysis, Table 2 assumes that the performance target for each evaluation attribute is equal to the best performance of either of the two competitors. For example, because higher levels of "pin capacity" are more preferable and the Sentry 50 has the highest level for this evaluation attribute, the Sentry 50 level (which is 256) is the performance target for this evaluation attribute. The targets for each evaluation attribute are shown in the second column of Table 2.

Keeney and Lilien (1987) used an additive measurable value function, and if the additive independence conditions in Theorem 1 hold, then the target-oriented preference function will have the weighted-additive form (8). Keeney and Lilien used a two-stage process to assess weights for a measurable value function. First, the relative weights for the evaluation attributes within each of the four evaluation categories were assessed, and then weights were assessed for each of the four categories so that the overall weight for each evaluation attribute is the product of its category weight and its within-category weight. To illustrate the target-oriented analysis procedure, Table 2 assumes that the weights used by Keeney and Lilien can be applied. Both the within-category weights and the category weights (which are 0.52, 0.14, 0.32, and 0.02) are shown in the third column of Table 2.

Using the performance data in Table 1 and the target and weight information in Columns 2 and 3 of Table 2, the fourth through sixth columns of Table 2 show whether each tester achieves the target on each evaluation attribute. For example, for "data rate," both the OR 9000 and Sentry 50 have a data rate of 50, which is the target level, and therefore they achieve the target, and hence have $p_i(x_i) = 1$

Table 2. New product application—target-oriented analysis.

			Target achievement			Weighted comparisons		
Evaluation attribute	Target	Weight	OR 9000	J941	Sentry 50	OR 9000	J941	Sentry 50
Technical		0.52				0.2	0.2	0.8
Pin capacity	256	0.15	0	0	1			
Vector depth	0.256	0.20	0	1	0			
Data rate	50	0.10	1	0	1			
Timing accuracy	600	0.35	0	0	1			
Pin capacitance	40	0.10	0	0	1			
Programmable measurement units	4	0.10	1	0	1			
Economic		0.14				0.5	1	0.5
Price	1	0.50	0	1	0			
Uptime	95	0.20	1	1	1			
Delivery time	6	0.30	1	1	1			
Software		0.32				1	1	1
Software translator	90	0.15	1	1	1			
Networking: Communications	1	0.20	1	1	1			
Networking: Open	0	0.20	1	1	1			
Development time	4	0.30	1	1	1			
Data analysis software	1	0.15	1	1	1			
Vendor Support		0.02				1	1	0.7
Vendor service	4.75	0.30	1	1	0			
Vendor performance	4	0.30	1	1	1			
Customer applications	1	0.40	1	1	1			
					Overall Value:	0.514	0.584	0.820

for this evaluation attribute. On the other hand, the J941 does not achieve the target with respect to this evaluation attribute, and hence has $p_i(x_i) = 0$. The weighted comparisons in the rightmost three columns of Table 2 show the weighted evaluation for each alternative within each of the four evaluation categories and the overall value for each alternative using the weights in the third column. This evaluation finds the OR 9000 to be the least preferred of the three alternatives, with an overall value of 0.514.

Both the Keeney and Lilien evaluation and this target-oriented analysis assume that there is no uncertainty about the performance of the alternatives. The target-oriented analysis extends to the case with uncertainty in a straightforward manner. Suppose first that the only uncertainty is with respect to the performance of the proposed OR 9000, and there is no uncertainty about the performance of the J941 and Sentry 50 because these are already in production. With an additive target-oriented preference function of the form of (8), only the marginal probability distributions for the evaluation attributes impact the ranking of alternatives. Hence, the probability analysis can be done one attribute at a time.

As an illustration, consider the "data rate" evaluation attribute, and represent this by Y. Let y_{J941} and $y_{\text{Sentry 50}}$ represent the data rates for the J941 and Sentry 50. Then, an alternative achieves the target for Y if y is at least as great as $\max(y_{\text{J941}}, y_{\text{Sentry 50}}) = 50$. Hence, the conditional probability of achieving the target for Y as a function of y is

$$p(I \mid y) = \begin{cases} 1, & y \geqslant 50, \\ 0, & \text{otherwise}. \end{cases}$$

If $f_{\text{OR 9000}}(y)$ and $F_{\text{OR 9000}}(y)$ represent the probability density function and cumulative distribution function, respectively, for $y_{\text{OR 9000}}$, then the expected value of the target-oriented preference function for data rate for the OR 9000 is

$$\begin{aligned} \mathrm{E}[u_{\text{data rate}} \mid \text{OR 9000}] &= \int_Y p(I \mid y) f_{\text{OR 9000}}(y)\, dy \\ &= 1 - F_{\text{OR 9000}}(50), \end{aligned}$$

which is known once $F_{\text{OR 9000}}(y)$ is assessed using any of the standard methods.

To illustrate a more complex case, suppose that the data rates for the J941 and Sentry 50 are also uncertain. Then, $p(I \mid y) = \mathrm{Prob}(y \geqslant \max(y_{\text{J941}}, y_{\text{Sentry 50}}) = \mathrm{Prob}(y \geqslant y_{\text{J941}} \cap y \geqslant y_{\text{Sentry 50}})$. If the data rates for these two testers are probabilistically independent, then $p(I \mid y) = \mathrm{Prob}(y \geqslant y_{\text{J941}}) \times \mathrm{Prob}(y \geqslant y_{\text{Sentry 50}})$, and the expected value for the OR 9000 of the target preference function for data rate is

$$\begin{aligned} &\mathrm{E}[u_{\text{data rate}} \mid \text{OR 9000}] \\ &\quad = \int_Y \int_{y\text{J941}=-\infty}^{y} \int_{y\text{Sentry 50}=-\infty}^{y} f_{\text{J941}}(y_{\text{J941}}) \\ &\qquad \cdot f_{\text{Sentry 50}}(y_{\text{Sentry 50}}) f_{\text{OR 9000}}(y)\, dy_{\text{Sentry 50}}\, dy_{\text{J941}}\, dy \\ &\quad = \int_Y F_{\text{Sentry 50}}(y) F_{\text{J941}}(y) f_{\text{OR 9000}}(y)\, dy, \end{aligned}$$

where the probability density functions and cumulative distribution functions for y_{J941} and $y_{\text{Sentry 50}}$ are represented analogously to those for $y_{\text{OR 9000}}$. It is straightforward to evaluate this integral using a standard approximation method such as the extended Pearson-Tukey approximation. (If the data rates for the J941 and Sentry 50 are not independent, then the probability model will be more complex, as would also be true in a realistic conventional multiattribute utility analysis for this situation.)

The Keeney and Lilien (1987) measurable value analysis requires ranges for all the evaluation attributes and midvalues for those ranges, while the target-oriented analysis requires that a target be specified for each attribute. Both methods require that attribute weights be assessed. The competitive shortcomings of the OR 9000 can be identified with either analysis approach, but these shortcomings are quickly apparent and easily understood from the target-oriented analysis results in Table 2. It is immediately clear from this table that the Sentry 50 has better technical performance than the OR 9000, and technical performance has a weight of 0.52, which is much greater than any other evaluation category. While either of the two analysis approaches can be a valid way to analyze this decision, the target-oriented approach seems easier to explain to a nontechnical audience, and it clearly emphasizes the differences among the alternatives.

7. Comparison to Multiattribute Utility Analysis

This section shows that common multiattribute utility function forms are strategically equivalent to various forms of target-oriented preference functions. However, in general there can be many target-oriented preference functions of a specified form that are strategically equivalent to a specified multiattribute utility function. For example, consider an additive multiattribute utility function $u(x) = \sum_{i=1}^{n} \lambda_i u_i(x_i)$. Let x_i^o represent the least-preferred x_i and x_i^* represent the most preferred x_i in the domain of interest, and assume that the $u_i(x_i)$ are scaled so that $u_i(x_i^o) = 0$ and $u_i(x_i^*) = 1$. Then, the additive multiattribute utility function can always be translated into a valid strategically equivalent additive target-oriented preference function $u_T(x) = \sum_{i=1}^{n} K_i p_i(x_i)$ by setting $p_i(x_i) = m_i^o + (m_i^* - m_i^o)u_i(x_i)$ and $K_i = \lambda_i/(m_i^* - m_i^o)$ for any constants $0 \leqslant m_i^o < m_i^* \leqslant 1$, where $m_i^o = p_i(x_i^o)$ and $m_i^* = p_i(x_i^*)$.

For example, consider the two-attribute additive utility function $u(x_1, x_2) = 0.4x_1 + 0.6(1 - x_2)$, where the domain for each X_i is $0 \leqslant x_i \leqslant 1$. One strategically equivalent target-oriented preference function is found by setting $m_i^o = 0$ and $m_i^* = 1$, $i = 1, 2$, to yield

$$u_T(x_1, x_2) = 0.4x_1 + 0.6(1 - x_2), \tag{9}$$

and another strategically equivalent target-oriented preference function is found by setting $m_1^o = 0.4$ and $m_1^* = 0.9$, while leaving $m_2^o = 0$ and $m_2^* = 1$, which yields

$$u_T(x_1, x_2) = 0.8 \times (0.4 + 0.5x_1) + 0.6(1 - x_2). \tag{10}$$

In (9), the probability of achieving the target for X_1 when $x_1 = 0$ is zero, and the probability when $x_1 = 1$ is one. However, in (10) the probability of achieving the target for X_1 when $x_1 = 0$ is 0.4, and the probability when $x_1 = 1$ is 0.9. Thus, the interpretation of (10) is different from (9), because in (9) there is a level of X_1 for which the target on X_1 is certain to be achieved and another level for which it is certain not to be achieved, but in (10) the probability of achieving the target on X_1 is never less than 0.4 or greater than 0.9. However, in (10) the utility K_1 of achieving the target for X_1 is twice as great relative to the utility K_2 of achieving the target for X_2 as it is in (9) (0.4 versus 0.6 in (9), but 0.8 versus 0.6 in (10)).

Theorems are now presented which demonstrate that target-oriented preference functions of the various types reviewed above can often be converted into strategically equivalent multiattribute utility functions, and vice versa. In these theorems, the notation defined above for x_i^o and x_i^* is used. Similarly, $x^o = (x_1^o, x_2^o, \ldots, x_n^o)$ and $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$. In presentations of multiattribute utility theory (Keeney and Raiffa 1976, Chapter 6; Kirkwood 1997, Chapter 9), the utility function is usually scaled so that $u(x^o) = 0$ and $u(x^*) = 1$, and for each x_i the single-attribute utility function $u_i(x_i)$ is scaled so that $u_i(x_i^o) = 0$ and $u_i(x_i^*) = 1$, and these scaling conventions are used here. Scaling constants for the single-attribute utility functions are designated by λ_i and the multiplicative utility function constant by λ.

Theorem 2. *There always exists a multilinear utility function*

$$u(x) = \sum_{i=1}^{n} \lambda_i u_i(x_i) + \sum_{i=1}^{n} \sum_{j>i} \lambda_{ij} u_i(x_i) u_j(x_j) + \sum_{i=1}^{n} \sum_{j>i} \sum_{l>j} \lambda_{ijl} u_i(x_i) u_j(x_j) u_l(x_l) + \cdots + \lambda_{123\cdots n} u_1(x_1) u_2(x_2) \cdots u_n(x_n)$$

that is strategically equivalent to any target-oriented preference function of form (6), *and vice versa.*

Theorem 3. *There always exists an additive utility function $u(x) = \sum_{i=1}^{n} \lambda_i u_i(x_i)$ that is strategically equivalent to any additive target-oriented preference function* (8), *and vice versa.*

Theorem 4. *There always exists*:

(1) *A multiplicative utility function $1 + \lambda u(x) = \prod_{i=1}^{n} [1 + \lambda \lambda_i u_i(x_i)]$, where $1 + \lambda = \prod_{i=1}^{n} (1 + \lambda \lambda_i)$ and $\lambda > 0$, that is strategically equivalent to any reliability-structured target-oriented preference function with series targets, provided that $p_i(x_i^o) > 0$ for all i,*

(2) *A reliability-structured target-oriented preference function with series targets and $p_i(x_i^o) > 0$ for all i that is strategically equivalent to any multiplicative utility function with $\lambda > 0$,*

(3) *A multiplicative utility function $1 + \lambda u(x) = \prod_{i=1}^{n} [1 + \lambda \lambda_i u_i(x_i)]$ with $1 + \lambda = \prod_{i=1}^{n} (1 + \lambda \lambda_i)$, where $-1 < \lambda < 0$, that is strategically equivalent to any reliability-structured target-oriented preference function with parallel targets, provided that $p_i(x_i^o) < 1$ for all i, and*

(4) *A reliability-structured target-oriented preference function with parallel targets and $p_i(x_i^o) < 1$ for all i that is*

strategically equivalent to any multiplicative utility function with $-1 < \lambda < 0$.

Theorem 4 shows that a multiplicative utility function with substitutable attributes ($-1 < \lambda < 0$) corresponds to a target-oriented preference function analogous to a parallel system in reliability theory, and a multiplicative preference function with complementary attributes ($0 < \lambda$) corresponds to a target-oriented preference function analogous to a series system in reliability theory. Because series and parallel systems are fundamental building blocks of reliability models and multiplicative utility functions are common in multiattribute utility applications, Theorem 4 shows that there is a close mathematical relationship between multiattribute utility theory and reliability theory.

8. Generalization: "Degree of Achievement" of Targets

This section generalizes the development in preceding sections to consider the "degree of achievement" of targets in a target-oriented preference function.

8.1. Single-Attribute Decisions

For a decision with a single evaluation attribute Z, let z_t be the possibly uncertain target level for Z and z_a be the possibly uncertain actual performance for alternative a, and assume that utility is specified as a function $u(z_t, z_a)$. For example,

$$u(z_t, z_a) = \begin{cases} 0, & z_a < z_t, \\ 1, & \text{otherwise}, \end{cases} \tag{11}$$

corresponds to a special case of §2.1 where preferences are increasing, and if $u(z_t, z_a)$ is a function only of z_a, this formulation corresponds to conventional utility analysis.

Note that $u(x_t, z_a)$ is not necessarily monotonic with respect to z_a in all practical decisions. For example, suppose

$$u(z_t, z_a) = \begin{cases} -a(z_t - z_a), & z_a < z_t, \\ b - c(z_a - z_t), & \text{otherwise}, \end{cases} \tag{12}$$

for constants $a \geqslant 0, b \geqslant 0$, and c. (Equation (11) is a special case of (12) with $a = 0$, $b = 1$, and $c = 0$.) In (12), if $a > 0$ there is added loss of value for missing the target z_t on the low side ($z_a < z_t$) by greater amounts, and either added value, no change in value, or added loss for exceeding the target z_t by greater amounts depending on whether $c < 0, c = 0$, or $c > 0$. For example, if $b = 0$, $a > 0$, and $c > 0$, then the most preferred level is $z_a = z_t$, and greater deviations from z_t in either direction are increasingly less preferred. Examples where this might hold include manufacturing processes where there is an "ideal" level for some characteristic of the product, materials management with a target inventory level, or medical conditions with an ideal level for a medical indicator, such as blood pressure.

General Formulation. If the probability density function for z_t and z_a, given a, is designated by $f_{t,a}(x_t, x_a \mid a)$, then

$$\mathrm{E}[u \mid a] = \int_{z_a=-\infty}^{\infty} \int_{z_t=-\infty}^{\infty} u(z_t, z_a) f_{t,a}(z_t, z_a \mid a)\, dz_t\, dz_a. \tag{13}$$

If z_t and z_a are probabilistically independent, given a, then $f_{t,a}(z_t, z_a \mid a) = f_t(z_t \mid a) \times f_a(z_a \mid a)$, and if z_t is also not dependent on a so that $f_{t,a}(z_t, z_a \mid a) = f_t(z_t) \times f_a(z_a \mid a)$, then (13) becomes

$$\mathrm{E}[u \mid a] = \int_{z_a=-\infty}^{\infty} f_a(z_a \mid a) \int_{z_t=-\infty}^{\infty} u(z_t, z_a) f_t(z_t)\, dz_t\, dz_a. \tag{14}$$

If $p(z_a) \equiv \int_{z_t=-\infty}^{\infty} u(z_t, z_a) f_t(z_t)\, dz_t$ in (14), then this equation is made equivalent to (1). (Because $u(z_t, z_a)$ can be rescaled by any positive affine transformation without changing the decision, it is always possible to specify $u(z_t, z_a)$ so that $p(z_a)$ gives valid probabilities.) Hence, there is always a target-oriented formulation (1) that is equivalent to (14), although that formulation may not have a natural interpretation in terms of the real-world decision.

As discussed in §2.1, there is always a standard single-attribute utility formulation that is strategically equivalent to any target-oriented specification of the form of (1). Because the preceding paragraph demonstrates that there is always a specification of the form of (1) that is strategically equivalent to any specification of the form of (14), therefore there is always a standard utility formulation that is strategically equivalent to (14).

8.2. Multiattribute Decisions

The approach in the preceding section can be generalized to multiattribute preferences, but utility independence concepts must be applied to develop a preference function form that is practical for applications. Designate the target level for attribute X_i by x_{it}, and the actual performance for X_i, given alternative a, by x_{ia}, and define $x_t \equiv (x_{1t}, x_{2t}, \ldots, x_{nt})$ and $x_a \equiv (x_{1a}, x_{2a}, \ldots, x_{na})$. Analogously to the single-attribute case in §8.1, assume that utility is a function $u(x_t; x_a)$. Designate the probability density function over x_t and x_a, given a, by $f_{t,a}(x_t; x_a \mid a)$ so that the multiattribute extension of (13) is

$$\mathrm{E}[u \mid a] = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} u(x_t; x_a)\, f_{t,a}(x_t; x_a \mid a)\, dx_t\, dx_a. \tag{15}$$

Reducing the complexity of (15) so that it can be applied requires simplifying both $u(x_t; x_a)$ and $f_{t,a}(x_t; x_a)$. To illustrate how this can be done, assume that the pairs $(X_{it}, X_{ia}), i = 1, 2, \ldots, n$, are each additive independent of the remaining X_{it} and X_{ia}. Then, $u(x_t; x_a)$ reduces to the standard weighted-sum form

$$u(x_t; x_a) = \sum_{i=1}^{n} k_i u_i(x_{it}, x_{ia}) \tag{16}$$

for some constants k_i and two-attribute utility functions $u_i(x_{it}, x_{ia})$. If, in addition, X_{it} is probabilistically independent of X_{ia}, and X_{it} is probabilistically independent of a, then

$$f_{t,a}(x_t; x_a \mid a) = f_t(x_t) f_a(x_a \mid a), \tag{17}$$

where $f_t(x_t)$ is the probability density function over the X_{it} and $f_a(x_a \mid a)$ is the conditional probability density function over X_{ia}, given a.

Substituting (16) and (17) into (15) and integrating yields

$$\mathrm{E}[u \mid a] = \sum_{i=1}^{n} k_i \int_{x_{ia}=-\infty}^{\infty} f_{ia}(x_{ia} \mid a) \cdot \int_{x_{it}=-\infty}^{\infty} u(x_{it}, x_{ia}) f_{it}(x_{it})\, dx_{it}\, dx_{ia}. \tag{18}$$

Because each term in (18) is analogous to (14), the argument that was applied to (14) also demonstrates that there is a strategically equivalent target-oriented formulation for (18) of the form of (8). As with (13), a form such as (12) could be used for the $u(x_{it}, x_{ia})$ in (18) to represent differing preferences for different degrees of achievement of the targets. Equation (18) assumes additive independence among the pairs (X_{it}, X_{ia}), but other preference conditions could also be applied, such as the utility independence and pairwise preferential independence conditions that lead to a multiplicative decomposition.

8.3. Relationship to Goal Programming

This section demonstrates that the basic weighted goal programming formulation is a special case of the model in §8.2. The basic weighted goal program is (Tamiz and Jones 1996)

$$\min_{y,\delta} \sum_{i=1}^{n} (w_i^- \delta_i^- + w_i^+ \delta_i^+) \tag{19}$$

subject to $f_i(y) + \delta_i^- - \delta_i^+ = b_i, i = 1, \ldots, n$, and $y \in C_y$. In this formulation, $b_i, i = 1, \ldots, n$, are the targets, δ represents the set of all $\delta_i^- \geqslant 0$ and $\delta_i^+ \geqslant 0$, which are the negative or positive deviation, respectively, from each target level, and $w_i^- \geqslant 0$ and $w_i^+ \geqslant 0$ are the weights for these deviations in the optimization. (At most, one of δ_i^- and δ_i^+ will be nonzero in any optimal solution.) C_y is an optional set of constraints on the decision variables $y = (y_1, y_2, \ldots, y_m)$. The solution to this goal program is the feasible y that minimizes the weighted sum of the deviations between $f_i(y)$ and b_i.

To show that this is a special case of generalized target-oriented preference analysis, assume (16) holds and (12) holds with $a \geqslant 0$, $b = 0$, and $c \geqslant 0$ for each $u_i(x_{it}, x_{ia})$ so that

$$u_i(x_{it}, x_{ia}) = \begin{cases} -a_i(x_{it} - x_{ia}), & x_{ia} < x_{it}, \\ -c_i(x_{ia} - x_{it}), & \text{otherwise.} \end{cases} \tag{20}$$

For this case, the target-oriented decision with no uncertainty can be represented by

$$\max_{x_a \in C} \sum_{i=1}^{n} k_i u_i(x_{it}, x_{ia}), \tag{21}$$

where C is the set of feasible $x_a = (x_{1a}, x_{2a}, \ldots, x_{na})$. By defining $\delta_i^- \equiv \max(x_{it} - x_{ia}, 0)$ to represent deviations below the target levels x_{it} in (20) and $\delta_i^+ \equiv \max(x_{ia} - x_{it}, 0)$ to represent deviations above the target levels in that equation, we can rewrite (20) as $u_i(x_{it}, x_{ia}) = -a\delta_i^- - c\delta_i^+$. Substituting this representation for u_i into (21) yields

$$\min_{\delta, x_a} \sum_{i=1}^{n} (k_i a_i \delta_i^- + k_i c_i \delta_i^+) \tag{22}$$

subject to $\delta_i^- - \delta_i^+ = x_{it} - x_{ia}$, $\delta_i^+ \geqslant 0$, $\delta_i^- \geqslant 0$, $i = 1, 2, \ldots, n$, and $x_a \in C$. If x_{ia} is a function of a set of decision variables y, so that $x_{ia} = f_i(y)$ with $y \in C_y$, and $b_i \equiv x_{it}$, we can rewrite $\delta_i^- - \delta_i^+ = x_{it} - x_{ia}$ as $f_i(y) + \delta_i^- - \delta_i^+ = b_i$. Hence, (22), which we have just shown is equivalent to (20) and (21), becomes equivalent to (19) if we set $w_i^- \equiv k_i a_i$ and $w_i^+ \equiv k_i c_i$. Because (16) and (20) are special cases of $u(x_t; x_a)$, the weighted goal programming formulation in (19) is a special case of target-oriented preference analysis.

Stochastic goal programming (Ballestero 2001) treats target-oriented decisions under uncertainty by replacing the uncertain evaluation attribute levels with their corresponding expected utilities and then minimizing (19), an approach that is not fully consistent with utility theory. In contrast, target-oriented preference analysis under uncertainty, as presented in §8.2, extends the goal programming formulation in (19) to decision making under uncertainty in a way that is fully consistent with utility theory.

8.4. Further Generalization to Target Ranges

The formulation in §§8.1 and 8.2 can be generalized to multiple target levels for each evaluation attribute, which addresses decisions with target *ranges*. Specifically, suppose there are two target levels $x_{it}^l < x_{it}^u$ for each X_i. If $x_{it} \equiv (x_{it}^l, x_{it}^u)$, then with the same utility and probabilistic independence conditions on X_{it} and X_{ia} as assumed in §8.2, (18) will be a valid representation for this situation. As an example, consider

$$u_i(x_{it}^l, x_{it}^u, x_{ia}) = \begin{cases} -a_i(x_{it}^l - x_{ia}), & x_{ia} < x_{it}^l, \\ 0, & x_{it}^l \leqslant x_{ia} \leqslant x_{it}^u, \\ -c_i(x_{ia} - x_{it}^u), & \text{otherwise,} \end{cases} \tag{23}$$

where $a_i > 0$ and $c_i > 0$, which assumes there is a *range* of levels $x_{it}^l \leqslant x_{ia} \leqslant x_{it}^u$ that are all equally preferred and deviations in either direction from that range are less preferred. (An example is a manufacturing process where any dimension for a manufactured component within a tolerance range is equally acceptable.) For this case, by analogous reasoning to that yielding (21), the target-oriented decision with no uncertainty can be represented as

$$\max_{x_a \in C} \sum_{i=1}^{n} k_i u_i(x_{it}^l, x_{it}^u, x_{ia}), \tag{24}$$

where C is the set of feasible $x_a = (x_{1a}, x_{2a}, \ldots, x_{na})$.

An analogous process to that in §8.3 develops a generalized version of (19) that is equivalent to (23) and (24). Define $\delta_i^{l-} \equiv \max(x_{it}^l - x_{ia}, 0)$ and $\delta_i^{l+} \equiv \max(x_{ia} - x_{it}^l, 0)$ to represent deviations from the lower target levels x_{it}^l in (23), and $\delta_i^{u-} \equiv \max(x_{it}^u - x_{ia}, 0)$ and $\delta_i^{u+} \equiv \max(x_{ia} - x_{it}^u, 0)$ to represent deviations from the upper target levels. Then, an equivalent formulation to (23) and (24) can be written as

$$\min_{\delta, x_a} \sum_{i=1}^{n} (k_i a_i \delta_i^{l-} + k_i c_i \delta_i^{u+}) \tag{25}$$

subject to $\delta_i^{l-} - \delta_i^{l+} = x_{it}^l - x_{ia}$, $\delta_i^{u-} - \delta_i^{u+} = x_{it}^u - x_{ia}$, $\delta_i^{l+} \geqslant 0$, $\delta_i^{l-} \geqslant 0$, $\delta_i^{u+} \geqslant 0$, $\delta_i^{u-} \geqslant 0$, $i = 1, 2, \ldots, n$, and $x_a \in C$, where at most one of the deviations will be nonzero for a specified i in any optimal solution. If x_{ia} is a function of a set of decision variables y, so that $x_{ia} = f_i(y)$ with $y \in C_y$, and $b_i \equiv x_{it}$, $w_i^- \equiv k_i a_i$, and $w_i^+ \equiv k_i c_i$, then (25) is made equivalent to a generalized form of (19). Because (23) and (24) are special cases of $u(x_t; x_a)$ with target ranges, the goal programming formulation in (25) is a special case of the generalized target-oriented preference formulation in this section.

9. Concluding Comments

This paper presents methods to model preference structures involving targets on multiple evaluation attributes. This approach can simplify the development of a multiattribute preference function for some decisions, and it appears to have particular applicability for situations where the outcome of the decision is significantly determined by the actions of other stakeholders to the decision. Examples of these types of decisions include new product development and decision making in a highly regulated environment.

Appendix

Following the notation of Keeney and Raiffa (1976), $\overline{X}_i$ stands for all the attributes except X_i, x_i' stands for an arbitrary specified level of X_i, x' stands for arbitrary specified levels of all the attributes, and $(x_i, \bar{x}_i')$ stands for any level of X_i combined with any specified levels for the other attributes.

Proof of Theorem 1. Consider two alternatives, a_1, which has a probability $1/n$ of yielding x and a probability $(n-1)/n$ of yielding x'; and a_2 which has a probability $1/n$ of yielding $(x_i, \bar{x}_i')$, $i = 1, 2, \ldots, n$. Because a_1 and a_2 have the same marginal probability distributions for each x_i, if the conditions of the theorem hold, then these two alternatives must be equally preferred and hence have equal expected values for u_T.

The expected value of u_T for a_1 is given by $(1/n) \cdot u_T(x) + [(n-1)/n] u_T(x')$, and the expected value of u_T for a_2 is given by $(1/n)\sum_{i=1}^n u_T(x_i, \bar{x}_i')$. Because the decision maker has independent targets, then $u_T(x)$ has the form of (6), and from inspection of (6)

$$\begin{aligned} u_T(x) &= a_i(\bar{x}_i) p_i(x_i) + b_i(\bar{x}_i)[1 - p_i(x_i)] \\ &= b_i(\bar{x}_i) + c_i(\bar{x}_i) p_i(x_i), \end{aligned} \tag{A-1}$$

for $i = 1, 2, \ldots, n$, for some functions $a_i(\bar{x}_i)$, $b_i(\bar{x}_i)$, and $c_i(\bar{x}_i) \equiv a_i(\bar{x}_i) - b_i(\bar{x}_i)$. (Note that $a_i(\bar{x}_i)$ and $b_i(\bar{x}_i)$ are not arbitrary because they are implicitly defined by (6).) Thus, the expected value of $u_T(x)$ for a_2 is equal to $(1/n)\sum_{i=1}^n [b_i(\bar{x}_i') + c_i(\bar{x}_i') p_i(x_i)]$. Equating this expression to the expression for the expected value of u_T for a_1 and rearranging terms leads to

$$u_T(x) = -(n-1) u_T(x') + \sum_{i=1}^n b_i(\bar{x}_i') + \sum_{i=1}^n c_i(\bar{x}_i') p_i(x_i).$$

Defining $K_i = c_i(\bar{x}_i')$ leads to (8) except for the constant $-(n-1)u_T(x') + \sum_{i=1}^n b_i(\bar{x}_i')$. However, two target-oriented preference functions that differ only by a constant are strategically equivalent. Therefore, this constant can be dropped from $u_T(x)$, and hence (8) follows except that we must establish that the K_i are nonnegative.

The nonnegativity of the K_i under the conditions of the theorem will be proved if $c_i(\bar{x}_i') = a_i(\bar{x}_i') - b_i(\bar{x}_i') \geqslant 0$, $i = 1, 2, \ldots, n$. Compare $a_i(\bar{x}_i')$ and $b_i(\bar{x}_i')$ in (A-1) with (6). From (6) it follows that $a_i(\bar{x}_i')$ and $b_i(\bar{x}_i')$ are made up of terms that are pairwise identical except for the constants k_I. Each pair of corresponding terms represents an outcome where the same targets are met except that for the term in $a_i(\bar{x}_i')$, the target for x_i is met in addition to the targets that are met for the corresponding term in $b_i(\bar{x}_i')$. Hence, from the statement of the theorem, the outcome for the term in $a_i(\bar{x}_i')$ must not be less preferred than the outcome for the corresponding term in $b_i(\bar{x}_i')$, and therefore the k_I for each term included in $a_i(\bar{x}_i')$ must be at least as great as the k_I for the corresponding term included in $b_i(\bar{x}_i')$. Hence, $K_i = c_i(\bar{x}_i') = a_i(\bar{x}_i') - b_i(\bar{x}_i') \geqslant 0$. □

For the following three proofs, it must be true that $u(x)$ and $u_i(x_i)$ are all scaled to lie between zero and one, but there is no such requirement on $u_T(x)$ or the $p_i(x_i)$. However, these can be rescaled to yield functions that are scaled between zero and one as follows:

$$u(x) \equiv [u_T(x) - u_T(x^o)]/[u_T(x^*) - u_T(x^o)], \tag{A-2}$$

$$u_i(x_i) \equiv [p_i(x_i) - p_i(x_i^o)]/[p_i(x_i^*) - p_i(x_i^o)]. \tag{A-3}$$

Because $u(x)$ as defined by (A-2) is a positive affine transformation of $u_T(x)$, then $u(x)$ and $u_T(x)$ are strategically equivalent.

Proof of Theorem 2. Keeney and Raiffa (1976, §6.5) have shown that if

$$\begin{aligned} u(x) = {} & u_i(x_i) u(x_i^*, \bar{x}_i) \\ & + [1 - u_i(x_i)] u(x_i^o, \bar{x}_i), \quad i = 1, 2, \ldots, n, \end{aligned} \tag{A-4}$$

then $u(x)$ has the multilinear form in Theorem 2. (Equation (A-4) is the same as (6.27) in Keeney and Raiffa (1976). Note that in this equation $u(x)$ is scaled so that $u(x^o)=0$ and $u(x^*)=1$ for some x^o and x^*, and also $u_i(x_i^o)=0$ and $u_i(x_i^*)=1$.) Therefore, showing that $u_T(x)$ in (6) is strategically equivalent to $u(x)$ in (A-4) will prove the theorem.

From (A-1) in the proof of Theorem 1, we know that $u_T(x) = b_i(\bar{x}_i) + c_i(\bar{x}_i)p_i(x_i), i = 1, 2, \ldots, n,$ for some functions $b_i(\bar{x}_i)$ and $c_i(\bar{x}_i)$. Solve (A-2) and (A-3) for $u_T(x)$ and $u_i(x_i)$, respectively, and substitute the resulting expressions into this equation. Rearranging terms leads to

$$u(x) = b_i'(\bar{x}_i) + c_i'(\bar{x}_i)u_i(x_i), \quad i = 1, 2, \ldots, n, \tag{A-5}$$

where $b_i'(\bar{x}_i) \equiv [-u_T(x^o) + b_i(\bar{x}_i) + c_i(\bar{x}_i)p_i(x_i^o)]/[u_T(x^*) - u_T(x^o)]$ and $c_i'(\bar{x}_i) \equiv c_i(\bar{x}_i) \times [p_i(x_i^*) - p_i(x_i^o)]/[u_T(x^*) - u_T(x^o)]$. Substituting first $x = (x_i^*, \bar{x}_i)$ and then $x = (x_i^o, \bar{x}_i)$ into (A-5) yields the equations $u(x_i^*, \bar{x}_i) = b_i'(\bar{x}_i) + c_i'(\bar{x}_i)u_i(x_i^*) = b_i'(\bar{x}_i) + c_i'(\bar{x}_i)$ and $u(x_i^o, \bar{x}_i) = b_i'(\bar{x}_i) + c_i'(\bar{x}_i)u_i(x_i^o) = b_i'(\bar{x}_i)$. Solving these for $b_i'(\bar{x}_i)$ and $c_i'(\bar{x}_i)$ and rearranging terms yields (A-4), and hence the result is proved.

Because a key step in this proof is not intuitively reversible, we will directly prove the converse of this theorem. (The step that is not intuitively reversible is going from (A-1) to (6).) We will proceed by inductively constructing a strategically equivalent target-oriented preference function (6) starting from any specified multilinear utility function. This proof requires some new notation. Specifically, define $y_k = (x_1, x_2, \ldots, x_k)$, $\bar{y}_k = (x_{k+1}, x_{k+2}, \ldots, x_n)$, and $I^k = (I_1, I_2, \ldots, I_k)$, where each I_i is a zero-one indicator variable. Then, I_u^k is defined to be the set of all 2^k combinations of possible levels of I^k. Finally, $y_k^{I^k}$ designates levels of $x_1, x_2, \ldots, x_k$ as follows: If $I_i^k = 1$ then $x_i = x_i^*$, and if $I_i^k = 0$ then $x_i = x_i^o$. For example, $y_3^{(0,1,1)} = (x_1^o, x_2^*, x_3^*)$.

The induction proceeds as follows: Assume that for a specific value of k

$$u(x) = \sum_{I \in I_u^k} u\big(y_k^{I^k}, \bar{y}_k\big) \left\{ \prod_{\substack{i=1 \\ i \ni (I_i^k = 1)}}^{k} u_i(x_i) \right\} \cdot \left\{ \prod_{\substack{i=1 \\ i \ni (I_i^k = 0)}}^{k} [1 - u_i(x_i)] \right\}. \tag{A-6}$$

(This is true from (A-4) for $k = 1$.) Then, apply (A-4) with $i = k+1$ to expand $u(y_k^{I^k}, \bar{y}_k)$ in (A-6). The result is

$$\begin{aligned} u(x) = {} & u_{k+1}(x_{k+1}) \sum_{I \in I_u^k} u\big(y_k^{I^k}, x_{k+1}^*, \bar{y}_{k+1}\big) \cdot \left\{ \prod_{\substack{i=1 \\ i \ni (I_i^k = 1)}}^{k+1} u_i(x_i) \right\} \left\{ \prod_{\substack{i=1 \\ i \ni (I_i^k = 0)}}^{k+1} [1 - u_i(x_i)] \right\} \\ & + [1 - u_{k+1}(x_{k+1})] \sum_{I \in I_u^k} u\big(y_k^{I^k}, x_{k+1}^o, \bar{y}_{k+1}\big) \cdot \left\{ \prod_{\substack{i=1 \\ i \ni (I_i^k = 1)}}^{k+1} u_i(x_i) \right\} \left\{ \prod_{\substack{i=1 \\ i \ni (I_i^k = 0)}}^{k+1} [1 - u_i(x_i)] \right\}. \end{aligned}$$

However, this can be rewritten as

$$u(x) = \sum_{I \in I_u^{k+1}} u\big(y_{k+1}^{I^{k+1}}, \bar{y}_{k+1}\big) \cdot \left\{ \prod_{\substack{i=1 \\ i \ni (I_i^{k+1} = 1)}}^{k+1} u_i(x_i) \right\} \left\{ \prod_{\substack{i=1 \\ i \ni (I_i^{k+1} = 0)}}^{k+1} [1 - u_i(x_i)] \right\},$$

and this is the same as (A-6) with k replaced by $k+1$. Hence, if (A-6) holds for k it also holds for $k+1$. When $k = n$, (A-6) is the same as (6) if we set $u(x) = u_T(x)$, $u(y_n^{I^n}, \bar{y}_{n+1}) = k_I$, and $p_i(x_i) = u_i(x_i)$ in (A-6). (When $k = n$, $\bar{y}_{n+1}$ is null, and hence $u(y_n^{I^n}, \bar{y}_{n+1})$ is equal to a constant.) Because these substitutions result in valid values for $u_T(x)$, k_I, and $p_i(x_i)$, we have constructed a strategically equivalent target-oriented preference function for the multilinear utility function, and thus the converse of the theorem is proved. □

Proof of Theorem 3. To show that there is always an additive utility function that is strategically equivalent to (8), solve (A-2) and (A-3) for $u_T(x)$ and $p_i(x_i)$, respectively, and substitute into (8). This yields

$$u_T(x^o) + [u_T(x^*) - u_T(x^o)]u(x) = \sum_{i=1}^{n} K_i\{p_i(x_i^o) + [p_i(x_i^*) - p_i(x_i^o)]u_i(x_i)\}.$$

Because $u_T(x^o) = \sum_{i=1}^n K_i p_i(x_i^o)$, this reduces to

$$[u_T(x^*) - u_T(x^o)]u(x) = \sum_{i=1}^{n} K_i[p_i(x_i^*) - p_i(x_i^o)]u_i(x_i).$$

Define $\lambda_i = K_i[p_i(x_i^*) - p_i(x_i^o)]/[u_T(x^*) - u_T(x^o)]$ and the result follows. If the λ_i do not sum to one, then multiply by the appropriate (positive) constant so this is true. (Multiplication by a positive constant always yields a strategically equivalent utility function.)

To show that there is always an additive target-oriented preference function (8) that is strategically equivalent to any additive utility function, substitute into the additive utility function $u(x) = \sum_{i=1}^n \lambda_i u_i(x_i)$ as follows: $u_T(x) = u(x)$, $p_i(x_i) = u_i(x_i)$, and $K_i = \lambda_i$. The result is a valid additive target-oriented preference function. □

Proof of Theorem 4. Each part of this theorem is proved in order. For Part (1) of the theorem, a reliability-structured target-oriented preference function with series targets has the form $u_T(x) = \prod_{i=1}^n p_i(x_i)$. Solving (A-2) and (A-3) for $u_T(x)$ and $u_i(x_i)$, respectively, and substituting into this equation yields

$$u_T(x^o) + [u_T(x^*) - u_T(x^o)]u(x) = \prod_{i=1}^{n}\{p_i(x_i^o) + [p_i(x_i^*) - p_i(x_i^o)]u_i(x_i)\}.$$

Define $\lambda = [u_T(x^*) - u_T(x^o)]/u_T(x^o)$ and $\lambda\lambda_i = [p_i(x_i^*) - p_i(x_i^o)]/p_i(x_i^o)$. By the conditions of the theorem $p_i(x_i^o) > 0$, and therefore $u_T(x^o) = \prod_{i=1}^n p_i(x_i^o) > 0$. Hence, both λ and λ_i must be greater than zero because $u_T(x^*) > u_T(x^o)$ and $p_i(x_i^*) > p_i(x_i^o)$. Substitute into the equation above to yield

$$u_T(x^o)[1+\lambda u(x)] = \left\{\prod_{i=1}^{n} p_i(x_i^o)\right\}\left\{\prod_{i=1}^{n}[1+\lambda\lambda_i u_i(x_i)]\right\}.$$

But because $u_T(x^o) = \prod_{i=1}^{n} p_i(x_i^o)$, therefore $1+\lambda u(x) = \prod_{i=1}^{n}[1+\lambda\lambda_i u_i(x_i)]$, which proves the result in Part (1). Each step of this proof is reversible, and so the converse of Part (1) is true, which establishes Part (2) of the theorem.

For Part (3) of the theorem, a reliability-structured target-oriented preference function with parallel targets has the form $1-u_T(x) = \prod_{i=1}^{n}[1-p_i(x_i)]$. Solving (A-2) and (A-3) for $u_T(x)$ and $u_i(x_i)$, respectively, and substituting into this equation yields

$$1-u_T(x^o)-[u_T(x^*)-u_T(x^o)]u(x) = \prod_{i=1}^{n}\{1-p_i(x_i^o)-[p_i(x_i^*)-p_i(x_i^o)]u_i(x_i)\}.$$

Define $\lambda = -[u_T(x^*)-u_T(x^o)]/[1-u_T(x^o)]$ and $\lambda\lambda_i = -[p_i(x_i^*)-p_i(x_i^o)]/[1-p_i(x_i^o)]$. By the conditions of the theorem, $p_i(x_i^o) < 1$, and therefore $1-u_T(x^o) = \prod_{i=1}^{n}[1-p_i(x_i^o)] > 0$. Because $u_T(x^*) > u_T(x^o)$, $p_i(x_i^o) < 1$, and $p_i(x_i^*) > p_i(x_i^o)$, then $-1 < \lambda < 0$ and $0 < \lambda_i$. Substitute into the equation above to yield

$$[1-u_T(x^o)][1+\lambda u(x)] = \left\{\prod_{i=1}^{n}[1-p_i(x_i^o)]\right\}\left\{\prod_{i=1}^{n}[1+\lambda\lambda_i u_i(x_i)]\right\}.$$

However, because $1-u_T(x^o) = \prod_{i=1}^{n}[1-p_i(x_i^o)]$, therefore $1+\lambda u(x) = \prod_{i=1}^{n}[1+\lambda\lambda_i u_i(x_i)]$, which proves the result in Part (3). Each step of this proof is reversible, and so the converse of Part (3) is true, which establishes Part (4) of the theorem. □

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