

Statistical Decisionmaking Without Math

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In his presidential address to the American Statistical Association in 1991, Vince Barabba emphasized that, to stay relevant, statistics needed to have an impact on executive decisionmaking in business and government. However, as statistical theory involves mathematics—which most executives either never learned or have forgotten—rigorous communication between statisticians and executives is difficult. Even established techniques for applying statistical decision theory to executive problems require knowledge of mathematical probability theory. As a result, trained decision consultants are usually required to do the required mathematical analysis privately and communicate the basic insights without mathematics. In this way, they help clients convince themselves of the proper course of action (versus trying to convince clients to accept the recommendations of a mathematical ‘analysis’).

But the vast majority of people, in addition to being unable to do the analyses themselves, cannot easily access decision consultants. To address this problem, we develop a purely visual method for applying decision theory that involves no mathematics. In developing this method, we are motivated by the success of the Venn diagram in making probability visual and the success many others have had in constructing visual proofs.

Elements of the Visual Approach

Visually Communicating Uncertainty Using Size

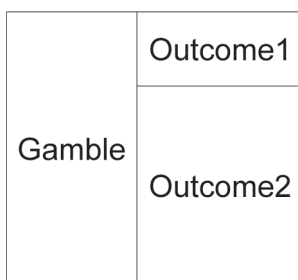


Figure 1.

We represent a gamble as a rectangle followed by two adjacent rectangles describing the possible outcomes of this gamble. The relative size of the adjacent rectangles for each outcome describes the chance of that outcome occurring. Thus, Figure 1 indicates the gamble is likely to lead to Outcome2, but does have some smaller chance of leading to Outcome1.



If we imagined throwing a dart at the two rectangles adjacent to the "Gamble" rectangle, the odds of the dart landing in the top rectangle (as opposed to the bottom rectangle) reflects the odds of having Outcome1 occur (as opposed to Outcome2.)

Now suppose Outcome2 was another gamble that was likely to lead to some payoff called "Outcome4," but had a small chance of leading to Outcome3. To show that Outcome2 is a gamble, Figure 1 would be replaced by Figure 2.

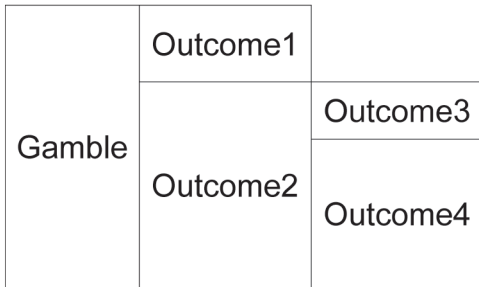


Figure 2.

While all outcomes are potentially affected by future uncertainties, we can usually identify outcomes that can be considered relatively certain (for the purposes of the decision we are making). Following L. J. Savage in his book *The Foundations of Statistics*, we call such outcomes "consequences."

Visually Communicating the Desirability of Gambles Using Color

We use the term "best possible" (and "worst possible") to describe the best possible (and worst possible) consequence that might arise from any of the decisions being considered. We define a basic gamble as a gamble that has some chance of leading to best possible and otherwise leads to worst possible. Figure 3 presents a specific basic gamble called "Risk."

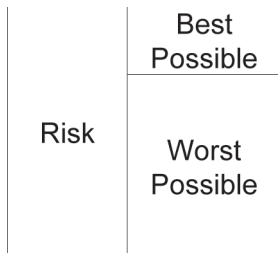


Figure 3.

We arbitrarily use white to color the rectangle for the best possible outcome and black to color the rectangle for the worst possible outcome. Coloring Figure 3 yields Figure 4.



Figure 4.

The next step is to color the "Risk" rectangle. To do so, imagine that the "Best Possible" and "Worst Possible" rectangles are cans of paint with the volume of each can being proportional to the rectangle's size. Imagine that both cans are emptied into the can represented by the "Risk" rectangle. This mixing of paints colors the "Risk" rectangle with the dark grey color shown in Figure 5. Note that changing the probability of the gamble leading to best possible adjusts the relative heights of the "Best Possible" rectangle and "Worst Possible" rectangle which, in turn, adjusts the color assigned to the "Risk" rectangle. This mixing procedure can assign colors to all possible basic gambles.



Figure 5.

For any consequence other than best possible and worst possible, we first identify a basic gamble that is just as desirable as that consequence. (This usually involves comparing the consequence with many basic gambles.) Our visual procedure colors this consequence with the same color as the gamble. Using this procedure, we assign colors to all possible consequences.

A visual proof

We now present a visual proof of an important result in decision theory. Consider the gamble in Figure 6a with two possible outcomes: Consequence 1 and Consequence 2.

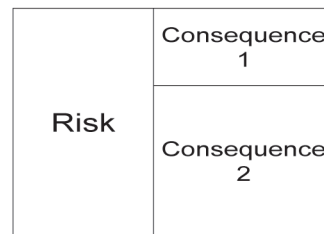


Figure 6a.

As both consequences are just as desirable as their corresponding basic gambles, we can rewrite the "Risk" by adding these corresponding gambles, as shown in Figure 6b.

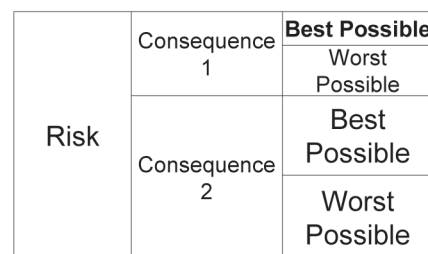


Figure 6b.

Because a decision's desirability is assumed to be determined by its final payoffs, we can erase the stack of intermediate outcomes and write the gamble as leading to the four outcomes shown in Figure 6c.



Figure 6c.

Rearranging the order in which the rectangles are listed gives Figure 6d.



Figure 6d.

Merging the two adjacent "Best Possible" rectangles and the two adjacent "Worst Possible" rectangles creates the basic gamble in Figure 6e (so that the "Risk" rectangle can be colored using the procedure for coloring basic gambles.) Thus, coloring gambles involving consequences is equivalent to coloring basic gambles. This justifies the coloring procedure presented in the text.

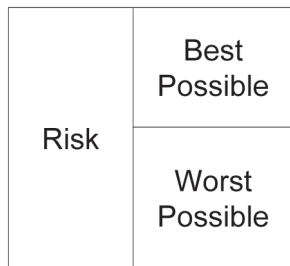


Figure 6e.

As the visual proof in Figure 6 shows, these assignments determine how colors should be assigned to all other possible gambles. As an illustration, consider the "Risk" rectangle shown in Figure 7. Color the rectangles for consequences 1 and 2. Just as was done for basic gambles, treat these two rectangles as cans of paint and 'pour' them into the "Risk" rectangle. This creates the appropriate color for the "Risk" rectangle. This 'mixing' procedure also is applicable to gambles with more than two outcomes.

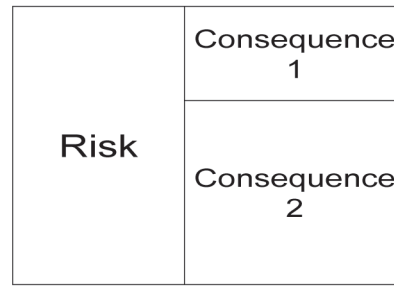


Figure 7.

Visually Describing Decisions

To distinguish between a decision and a gamble, we represent a decision by a rectangle connected by arcs to the possible choices for the decision (see Figure 8).

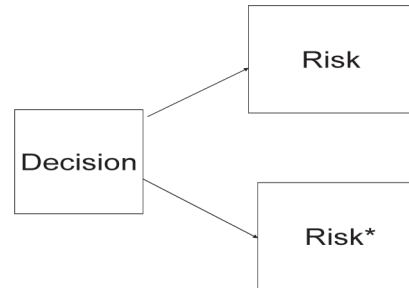


Figure 8.

As Figure 9 shows, we then color the outcomes of the decision. Now if a decision were like a risk, the decision rectangle would be colored by mixing colors from the "Risk" rectangle and the "Risk*" rectangle. But in a decision, we can choose which outcome happens. So we choose to make the better (lighter) outcome, "Risk," happen. We then color the "Decision" rectangle with the light grey color of the "Risk" rectangle (see Figure 10.)

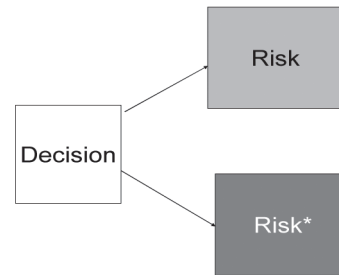


Figure 9.

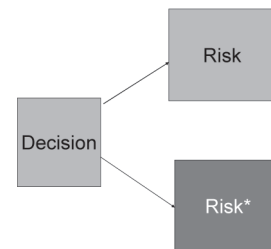


Figure 10.

Demonstration of Visual Decision Analysis

A Textbook Decision Problem

Consider a risky project with a small chance of achieving its goals and a large chance of failure. If the project achieves its goals, there is an even chance of the project having a high payoff versus a low payoff. You must decide whether to invest resources in the project. (Both payoffs and resources could be nonmonetary as well as monetary.) Thus,

You are first confronted with a decision (e.g., whether to do the project)

You then make that decision

If you decide to do the project, you learn whether the project is successful

If the project succeeds, you learn whether the payoffs will be high or low

To construct the visual representation, draw a sequence of stacked rectangles in the order in which the corresponding outcome occurs, with the relative size of a rectangle within a stack describing the chance of the event occurring (see Figure 11).

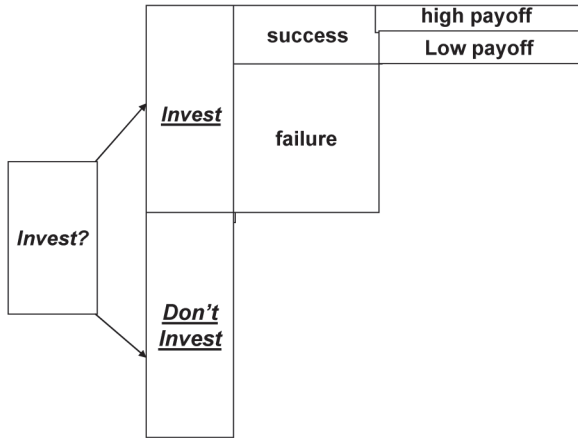


Figure 11.

To determine the best decision, you first assess the relative desirability of the rightmost outcomes: "Don't Invest," "failure," "low payoff," and "high payoff." The best outcome, "high payoff," has its rectangle colored white, while the worst outcome, "failure," has its rectangle colored black. Other outcomes are given intermediate colors (see Figure 12). (Technically, these assessed colors for the end rectangles correspond to the 'utility,' or 'loss,' of getting these outcomes as payoffs.) We now discuss how this color assignment leads to colors for all the other rectangles.

Note that we cannot yet specify how to color the rectangle "Invest?" as it is followed by the rectangle, "Invest," which has not been assigned a color. But we can specify how to color the rectangle "success," as it is followed by two rectangles, "high payoff" and "low payoff," which have been colored. Because the two rectangles are outcomes of risk, we color the "success" rectangle by mixing the colors of the "high payoff" and "low payoff" rectangles. This gives Figure 13.

Now consider the remaining uncolored rectangles. Note that the "Invest" rectangle is now followed by two colored

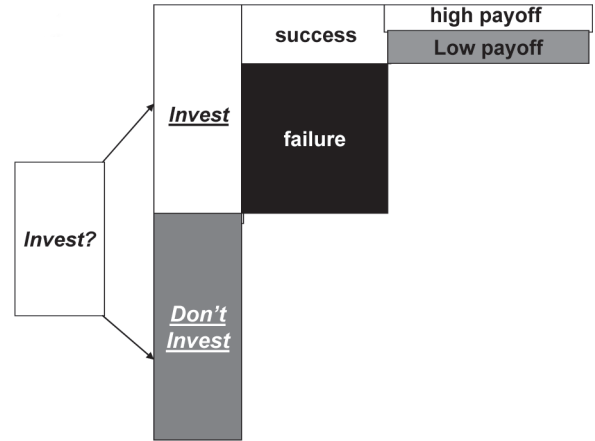


Figure 12.

rectangles. Because the two colored rectangles, "failure" and "success," are risks, we can color the "Invest" rectangle by mixing their colors. As the "failure" rectangle is bigger than the "success" rectangle, this gives Figure 14. (Technically, the color assigned to Invest is just the utility of getting the high payoff, weighted by the probability of getting high payoff, plus the utility of getting low payoff, weighted by the probability of getting low payoff, plus the utility of failure, weighted by the probability of failure. This is called the expected utility or expected loss of Invest.)

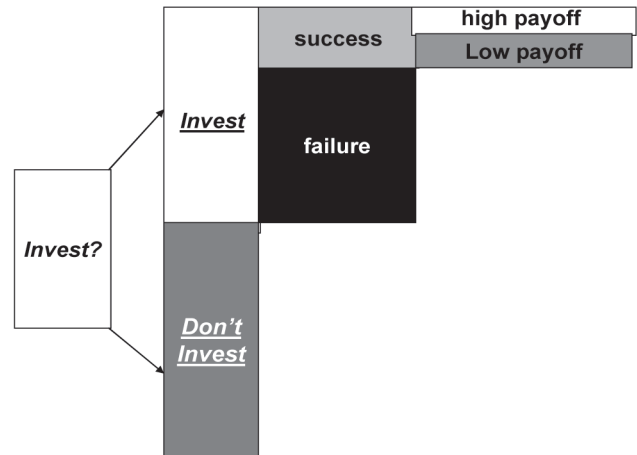


Figure 13.

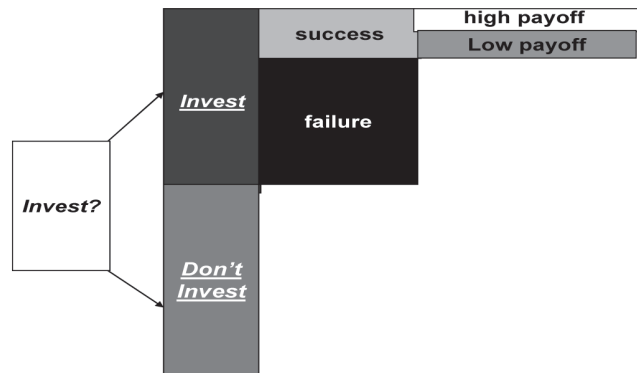


Figure 14.

This leaves only one uncolored rectangle, "Invest?". Since this rectangle is now followed by two colored rectangles, "Invest" and "Don't Invest," which are decision outcomes, we make the decision Don't Invest, as it has the lighter color, and color Invest? with this lighter color (see Figure 15). (This corresponds to the central theorem of decision analysis: A rational individual always chooses that decision with the highest expected utility or the smallest expected loss.)

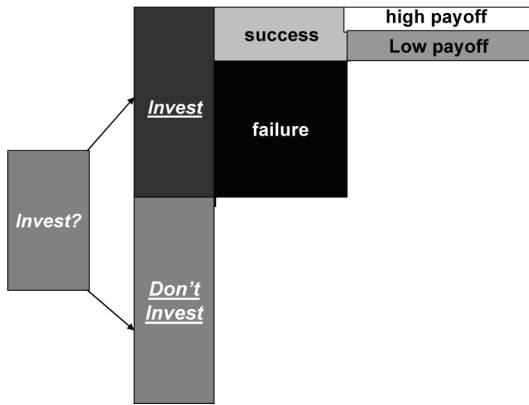


Figure 15.

The Textbook Decision Problem with Perfect Information

For a more complicated example, consider the previous problem where we first learn the outcome of one of our uncertainties (i.e., the potential payoffs of a successful project) before making a decision. In this case, the following would take place:

You begin with uncertainty about the payoff of the potential project

You then learn what the potential payoff of a successful project could be

You then decide whether to do the project

You then learn whether the project was successful

The visual formalization of this problem is shown in Figure 16. As before, "high payoff" and "low payoff" have the same-sized rectangles, while the "failure" rectangle is much larger than the "success" rectangle.

The next step involves coloring the rightmost rectangles using information from the original diagram. Two of the end

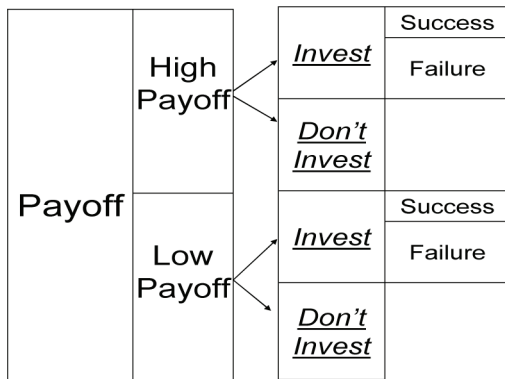


Figure 16.

rectangles in this new diagram, "Don't Invest" and "Failure," were end rectangles in the original diagram. To be consistent with this original diagram, we use dark grey to color both of the "Don't Invest" rectangles and black to color both of the "Failure" rectangles.

But the outcome, "Success," was not an end rectangle in the original diagram. To determine its color, note that the uppermost success event presumes a high project payoff. Because a successful project with a high project payoff led to a white rectangle in the original diagram, we color this uppermost "Success" rectangle white (see Figure 17). In contrast, the lowermost "Success" event presumes a low project payoff. Because a successful project with a low payoff led to a light grey rectangle in the original diagram, we use light grey to color this second "Success" rectangle. The colors assigned to these end rectangles will now be used to infer colors for all the other rectangles.

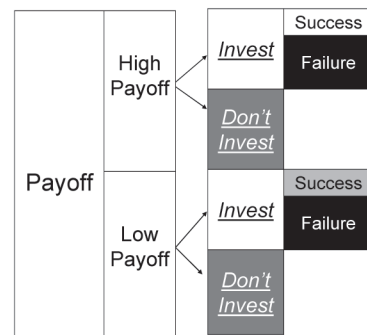


Figure 17.

The next rectangles that can be colored are the "Invest" rectangles, which in both cases are followed by equally sized "Success" and "Failure" chance rectangles. Mixing colors appropriately gives Figure 18.

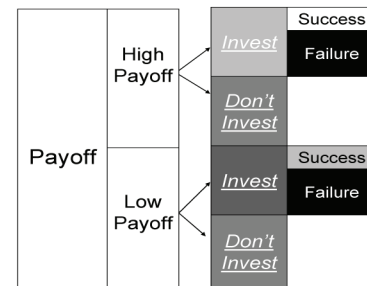


Figure 18.

We then color the "high payoff" rectangle, which borders two decision rectangles (italicized and underlined). Because the lighter rectangle is "Invest," you choose to invest (if you learn the payoff is high) and color the "high payoff" rectangle light grey (see Figure 19). To color the "low payoff" rectangle, note that the lighter-colored rectangle is "Don't Invest." So, you decide not to invest (if you learn the payoff is low) and color the "low payoff" rectangle an intermediate dark grey.

Finally, we color the project "payoff" rectangle by mixing the color of the two chance rectangles, "high payoff" and "low payoff" (see Figure 20). Note that you only change your

original decision (to not invest) if the information indicates the project payoff is high.

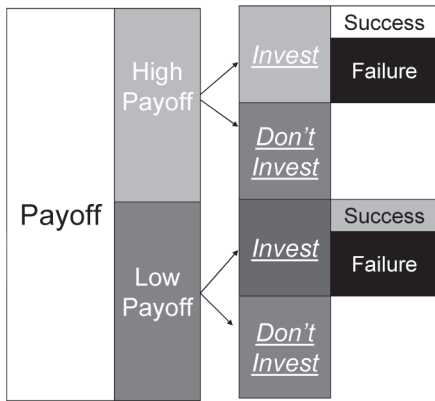


Figure 19.

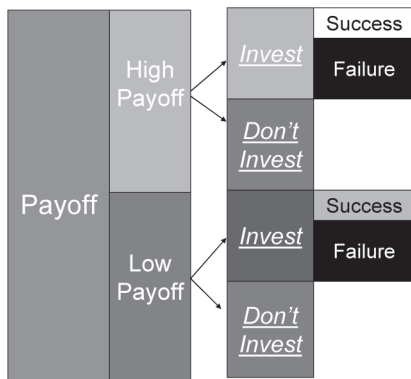


Figure 20.

We now compare the starting rectangle for the original problem (without perfect information) and this modified problem (with perfect information). Note that the rectangle for the problem with perfect information is lighter (see Figure 21), indicating it is more desirable. To quantify how much more valuable the second problem is, suppose we altered the modified problem by requiring the individual pay a fee for the perfect information. Incorporating this fee would make all the consequences of the modified problem less desirable, and thus would darken all the end rectangles. We could solve the problem with this modification and determine the new color attached to the starting rectangle with the fee for perfect

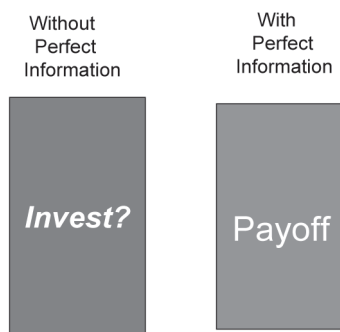


Figure 21.

information. When the fee is large enough, the color of the starting rectangle will match the color of the original problem. The fee will represent the value of perfect information.

Conclusions

Visualization is widely accepted as a useful technique for making statistical results more transparent. But making the results of the analysis (and even the input data) transparent does little good if the user does not understand and distrusts the analysis that led to those results. We showed how the entire analysis (as well as input data and results) could be made visual. Hence, it makes decision analysis (and thus statistical methods) more accessible to a wider audience.

Our visual approach does have certain limitations. While there will be a visual difference between an event of probability 0.25 and an event of probability 0.26, most individuals will not be able to discern such a small difference in our diagrams. In many problems, this lack of precision is acceptable. But there will be some problems where precision is important. (For example, a risk leading to a 1% chance of death might be treated very differently than a risk with a 1 in a 1,000 chance of death.) When confronted with these kinds of problems, individuals probably do need to consult a quantitative expert.

However, an individual who uses our approach for decisions in the many contexts where imprecision is acceptable will certainly become more familiar with statistical thinking. In becoming a "visual statistician," this individual also will have a better comprehension of the analysis of a statistical expert on those problems where precision is essential. And this may lead to an improved acceptance of expert statistical analyses. 📺

Further Reading

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